



Manonmaniam Sundaranar University, Directorate of Distance & Continuing Education, Tirunelveli

***Manonmaniam Sundaranar University,
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OPEN AND DISTANCE LEARNING (ODL) PROGRAMMES

(FOR THOSE WHO JOINED THE PROGRAMMES FROM THE ACADEMIC YEAR 2023–2024)

B.Sc. Physics

III Year

ATOMIC AND NUCLEAR PHYSICS

Course Material

Prepared

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ATOMIC AND NUCLEAR PHYSICS

UNIT-I

VECTOR ATOM MODEL: Introduction – Vector atom model – electron spin – spatial quantisation –quantum numbers associated with vector atom model. Coupling Schemes, L-S and j-j coupling. Pauli's Exclusion Principle. Magnetic dipole moment due to orbital motion and spin motion of the electron –Bohr magnetron –Stern-Gerlach experiment

UNIT-II

ATOMIC SPECTRA : Spectral terms and notations –Zeeman Effect –quantum mechanical explanation of normal Zeeman effect. Anomalous Zeeman Effect–quantum mechanical theory– fine structure of sodium D-lines. Paschen-Back Effect, Stark Effect.

UNIT-III

STRUCTURE OF NUCLEI: General Nuclear Properties –charge, size, shape, mass, density, spin, parity. Mass defect and Binding energy – Binding energy curve. Nuclear force – characteristics of nuclear forces. Nuclear Models – Liquid Drop Model –similarities between nucleus and liquid drop –mass formula. Shell Model –magic numbers–evidences that led to shell

UNIT-IV

RADIOACTIVITY: Discovery of radioactivity – exponential decay law half-life, mean-life. Natural and Artificial radio activity. Properties of alpha rays, beta rays and gamma rays – Gamow's theory of alpha decay (qualitative study) – Geiger-Nuttall law –beta decay spectra

UNIT-V

Nuclear Reactor, Accelerator and Detectors: Nuclear fusion–Nuclear fission. Nuclear Reactor – construction and working –radio isotopes and its applications. Charged Particle Accelerators - Cyclotron –Detectors of Nuclear Radiation –Geiger Muller Counter –Scintillation Counter .Nuclear facilities in India.



Unit 1: Vector Atom Model

1. Introduction
2. Electron Spin
3. Spatial Quantisation
4. Quantum numbers associated with vector atom model
5. L-S coupling
6. J-J coupling
7. Pauli's Exclusion Principle
8. Magnetic Dipole moment due to orbital motion and spin motion of the electron
9. Bohr Magnetron
10. Stern-Gerlach Experiment

Introduction

The Vector Atom Model, proposed by Sommerfeld, is an improved version of Bohr's atomic model. While Bohr considered electrons to revolve in fixed circular orbits, his model could not explain phenomena such as the fine structure of spectral lines, the Zeeman effect (splitting of spectral lines in a magnetic field), or the Stark effect (splitting in an electric field). To address these limitations, the Vector Atom Model introduced the idea that an electron possesses multiple types of angular momenta, such as orbital angular momentum and spin angular momentum, each represented as a vector with both magnitude and direction. These vectors combine to form the total angular momentum of the electron. By accounting for the vector nature of these angular momenta, the model successfully explained many spectral features that Bohr's model could not, and it also laid the foundation for the development of



modern quantum mechanics. These angular momenta must be treated as **vectors**, not simple numbers.

Sommerfeld's model thus incorporated:

1. **Orbital angular momentum**
2. **Spin angular momentum** (explained later by Uhlenbeck and Goudsmit in 1925)
3. **Total angular momentum**

This model became an important bridge between Bohr's classical model and modern quantum mechanics.

Applications / Achievements of the Vector Atom Model

The Vector Atom Model successfully explained several important observations:

(i) Fine Structure of Spectral Lines

The splitting of hydrogen lines into closely spaced components is explained through spin-orbit coupling.

(ii) Zeeman Effect

The splitting of spectral lines when an atom is placed in a magnetic field is explained by quantized orientations of angular momentum vectors.

(iii) Stark Effect

The splitting of lines in an electric field is understood through different vector orientations and energies.

(iv) Introduction of Quantum Numbers

The model reinforced and clarified the role of the quantum numbers:



- Principal (n)
- Azimuthal (l)
- Magnetic (m_l)
- Spin (m_s)

(v) Foundation for Quantum Mechanics

The model's vector treatment of angular momenta became the basis for the later quantum mechanical model proposed by Schrödinger and Heisenberg.

Below is a simple conceptual representation of how angular momentum vectors exist in the atom:

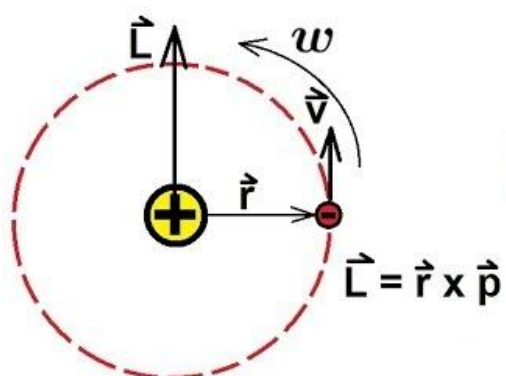


Figure 1.1

- \vec{L} points in a certain direction due to the electron's orbital motion.
- \vec{r} points differently due to the electron's spin.
- $\vec{\omega}$ is the resultant vector obtained by combining \vec{L} and \vec{r} .

$$\vec{L} = \vec{r} \times \vec{p}$$

The quantization of the directions of these vectors (in a magnetic field) explains the splitting of spectral lines.

Limitations of the Vector Atom Model



1. Could not fully explain spectra of multi-electron atoms

The model works reasonably well for hydrogen but fails when electron–electron repulsion becomes significant in larger atoms.

2. Incomplete explanation of the anomalous Zeeman effect

While it explains the normal Zeeman effect, the splitting observed in many atoms is more complex and cannot be accounted for by this model.

3. Does not account accurately for intensity of spectral lines

The model can predict the number of lines but not their relative intensities or probabilities.

4. Incomplete understanding of electron spin at the time

Spin was incorporated later but without a fully correct quantum-mechanical interpretation, making the model partially inconsistent.

5. Could not explain hyperfine structure

Splitting caused by interactions between electron angular momentum and nuclear spin remained unexplained.

6. Fails to describe wave nature of electrons

The model still treats electrons as particles in fixed orbits; it does not include de Broglie's wave concept or Schrödinger's wave mechanics.

7. No explanation for orbital shapes

It cannot describe s, p, d, and f orbital shapes or the probability distribution of electrons.

8. Relies partly on classical ideas

Important aspects like vector addition of angular momenta were combined with classical orbit ideas, leading to internal contradictions.



Electron Spin

Electron spin is an **intrinsic property** of the electron. It is not due to any physical rotation of the electron, but it behaves *as if* the electron is spinning on its own axis.

This spin gives rise to:

- **Spin angular momentum**
- **Spin magnetic moment**

Both are essential to explain the fine splitting of spectral lines.

Spin was not originally part of Sommerfeld's work but was introduced in **1925** by **Uhlenbeck and Goudsmit** and later incorporated into the Vector Atom Model.

Spin Angular Momentum

Every electron has a fixed spin quantum number:

$$s = 1/2$$

The magnitude of spin angular momentum is given by:

$$S = \sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}\left(\frac{3}{2}\right)}\hbar$$

This value is the same for all electrons.

Spin Magnetic Moment

The electron is charged, so its spin produces a magnetic moment.

This magnetic moment is stronger than the magnetic moment from orbital motion.

It is proportional to:

$$\mu_s \propto \sqrt{s(s+1)}$$

This magnetic effect is essential in explaining:

- **Fine structure**
- **Anomalous Zeeman effect**

Spin Projection (m_s)

The spin angular momentum can be oriented in two possible ways along a magnetic field axis.

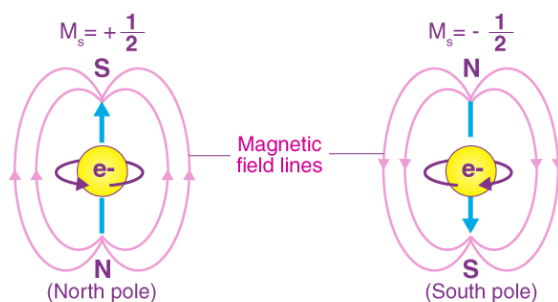


Figure 1. 1

The spin magnetic quantum number is:

- $m_s = +1/2 \rightarrow$ “spin up”
- $m_s = -1/2 \rightarrow$ “spin down”

This two-valued property was the basis of Pauli’s Exclusion Principle.

Role of Spin in the Vector Atom Model

Electron spin plays a key role in:



(i) Spin–Orbit Coupling

The electron's spin interacts with its orbital motion.

This interaction causes **small energy shifts**, producing **fine structure** in spectral lines.

(ii) Total Angular Momentum (**J**)

In the Vector Atom Model, the **total angular momentum** is formed by vector addition:

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

where,

- **L** = orbital angular momentum
- **S** = spin angular momentum

Because they are vectors:

- Their directions matter
- Their magnitudes combine quantum mechanically

This produces multiple possible values of **J**, which leads to splitting of energy levels.

(iii) Explanation of Zeeman and Stark Effects

The split spectral lines arise due to different orientations of the total angular momentum vector, which depends on spin.

Spatial Quantisation

Spatial quantisation refers to the phenomenon where the angular momentum vector of an electron can orient itself only in certain fixed (discrete) directions in space when placed in an external magnetic field. In classical physics, a rotating body (like a spinning top) can orient its angular momentum in any direction. But in an atom, the electron's angular momentum is quantised, meaning it can take only specific allowed orientations. This



restriction of direction is known as spatial quantisation. Spatial quantisation arises due to the **quantisation of angular momentum**, a key postulate of quantum theory.

In the Vector Atom Model, the electron has:

- **Orbital angular momentum (L)**
- **Spin angular momentum (S)**
- **Total angular momentum (J)**

All these angular momenta have:

- **a fixed magnitude**, and
- **only certain allowed directions** in space.

The direction is determined by the **magnetic quantum number (m_l)** for L or **m_j** for J.

Quantisation of Angular Momentum Direction

Magnitude of orbital angular momentum

$$L = \sqrt{l(l+1)}\hbar$$

Component along a magnetic field (Z-axis)

$$L_z = m_l \hbar$$

Here:

- $l = 0, 1, 2, \dots$
- $m_l = -l, -(l-1), \dots, 0, \dots, +(l-1), +l$

Thus **m_l takes only $2l + 1$ discrete values** \rightarrow

$2l + 1$ possible orientations for L. This is spatial quantisation.

When an atom is placed in a magnetic field:

- The angular momentum vector **cannot rotate freely**.



- It makes a fixed angle with the field direction (Z-axis).
- Only certain *tilted* positions are allowed.

Thus the vector behaves like a spinning top whose axis can take only fixed orientations.

The Stern–Gerlach Experiment (1922)

This is the most famous proof of spatial quantisation.

- A beam of silver atoms was passed through a non-uniform magnetic field.
- The beam **split into discrete parts** instead of forming a continuous pattern.
- This showed that the electron's angular momentum has **only certain quantised orientations**.

For silver (with electron spin $s = 1/2$), the beam split into **two parts**, proving $m_s = \pm 1/2$.

This confirmed spatial quantisation experimentally.

Spatial Quantisation and Spectral Lines

Spatial quantisation explains:

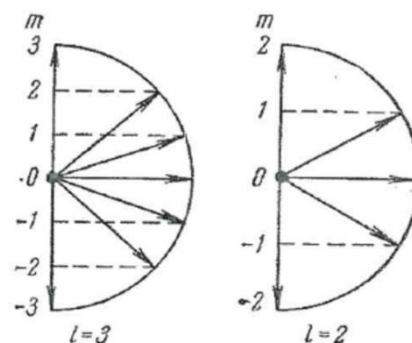
(i) Zeeman Effect

- In a magnetic field, energy levels split depending on allowed m_l values.
- This produces multiple spectral lines.

(ii) Fine Structure

- Different orientations of L and S cause slight energy differences.

(iii) Stark Effect





- Electric field also causes splitting due to orientation-dependent energy changes.

Thus spatial quantisation is crucial for understanding atomic spectra.

Quantum numbers associated with vector atom model

In the Vector Atom Model, electrons in an atom are described using **four fundamental quantum numbers**. These quantum numbers arise because the electron possesses **quantised angular momenta** (orbital, spin, and total) and corresponding magnetic effects.

These quantum numbers determine:

- Size of the orbit
- Shape of the orbital
- Orientation of angular momentum
- Spin behaviour of the electron
- Total angular momentum

1. Principal Quantum Number (n)

Symbol: n

Values: $n = 1, 2, 3, \dots$

Meaning:

- Determines the main energy level or shell of the electron.
- Larger n means bigger orbit and higher energy.
- Corresponds to K, L, M, N... shells.

Role in Vector Atom Model:

Gives the number of possible sublevels within each shell and influences the total angular momentum values.

2. Azimuthal (Orbital) Quantum Number (l)



Symbol: l

Values:

$$l = 0, 1, 2, \dots, (n - 1)$$

Meaning:

- Determines the shape of the orbital.
- Determines magnitude of orbital angular momentum:

$$L = \sqrt{l(l + 1)}\hbar$$

Subshell Notation:

- $l = 0 \rightarrow s$
- $l = 1 \rightarrow p$
- $l = 2 \rightarrow d$
- $l = 3 \rightarrow f$

Role in Vector Atom Model:

Defines one of the key angular momentum vectors (L).

3. Magnetic Quantum Number (m_l)

Symbol: m_l

Values:

$$m_l = -l, -(l - 1), \dots, 0, \dots, +(l - 1), +l$$

Meaning:

- Determines the orientation of the orbital angular momentum vector L in space.
- Total number of possible orientations = $2l + 1$.



Component of Angular Momentum:

$$L_z = m_l \hbar$$

Role in Vector Atom Model:

Explains spatial quantisation, Zeeman effect, and splitting of spectral lines.

4. Spin Quantum Number (s)

Symbol: s

Value for electron:

$$s = \frac{1}{2}$$

Meaning:

- Represents the intrinsic spin angular momentum of the electron.
- Magnitude of spin angular momentum:

$$S = \sqrt{s(s+1)}\hbar$$

Role in Vector Atom Model:

Introduces the concept of spin magnetic moment, essential for fine-structure and anomalous Zeeman effect.

5. Spin Magnetic Quantum Number (m_s)

Symbol: m_s

Values:

$$m_s = +\frac{1}{2}, -\frac{1}{2}$$



Meaning:

- Indicates the orientation of the electron's spin.
- Explains splitting into two beams in the Stern–Gerlach experiment.

6. Total Angular Momentum Quantum Number (j)

Symbol: j

Values:

$$j = l \pm s$$

Since $s = 1/2$:

$$j = l + \frac{1}{2}, j = l - \frac{1}{2}$$

Meaning:

- Represents the magnitude of total angular momentum (J):

$$J = \sqrt{j(j+1)}\hbar$$

Role in Vector Atom Model:

Essential to describe fine structure, spin–orbit coupling, and energy level splitting.

7. Total Magnetic Quantum Number (m_j)

Symbol: m_j

Values:

$$m_j = -j, -(j-1), \dots, 0, \dots, +(j-1), +j$$

Meaning:

- Gives the number of possible orientations of total angular momentum in space.



- Determines the splitting of levels under external fields (Zeeman and Stark effects).

Quantum Number	Symbol	Values	Determines
Principal	n	1,2,3...	Energy level (shell)
Azimuthal	l	$0 \rightarrow n-1$	Orbital shape, orbital angular momentum
Magnetic	m_l	$-l \rightarrow +l$	Orientation of orbital angular momentum
Spin	s	$1/2$	Intrinsic spin of electron
Spin magnetic	m_s	$\pm 1/2$	Orientation of spin
Total angular momentum	j	$l \pm 1/2$	Magnitude of total angular momentum J
Total magnetic	m_j	$-j \rightarrow +j$	Orientation of total angular momentum

L-S coupling

L–S coupling, also known as Russell–Saunders coupling, is one of the fundamental schemes used to describe the interaction between the orbital and spin angular momenta of electrons inside an atom. It is particularly applicable to light atoms (low atomic number), where electrostatic interactions between electrons dominate over spin–orbit interactions.

In multi-electron atoms, each electron possesses two types of angular momentum: orbital angular momentum (l) due to its motion around the nucleus, and spin angular momentum (s) due to its intrinsic spin. Instead of coupling the orbital and spin angular momentum of each electron individually, L–S coupling first combines all orbital angular momenta to form a resultant total orbital angular momentum L , and all spin angular momenta to form a resultant total spin angular momentum S . These two resultant vectors then couple together to form the total angular momentum J of the atom.



This coupling scheme helps in predicting the fine structure of atomic spectra, allowed transitions, energy levels, and term symbols. It is widely used in quantum mechanics to interpret spectral lines of lighter elements where spin–orbit interaction is relatively weak compared to electron–electron repulsion.

L–S coupling is the appropriate coupling scheme for **light atoms** (small atomic number Z), where:

- **Electrostatic (electron–electron) interactions** are *stronger* than the spin–orbit interaction, and
- Individual orbital angular momenta \mathbf{l}_i couple together to give a total orbital angular momentum \mathbf{L} , and individual spins \mathbf{s}_i couple together to give a total spin \mathbf{S} .

Finally \mathbf{L} and \mathbf{S} combine to give the total angular momentum \mathbf{J} :

$$\mathbf{L} = \sum_i \mathbf{l}_i, \quad \mathbf{S} = \sum_i \mathbf{s}_i, \quad \mathbf{J} = \mathbf{L} + \mathbf{S}.$$

A term is written as

where

- $2S + 1$ is the **multiplicity** (number of spin states),
- L is the spectroscopic letter corresponding to the total orbital angular momentum quantum number L (i.e. $L = 0 \rightarrow S, 1 \rightarrow P, 2 \rightarrow D, 3 \rightarrow F, \dots$),
- J is the total angular momentum quantum number with allowed values

$$J = |L - S|, |L - S| + 1, \dots, L + S.$$



Multiplicity $2S + 1$ gives the number of sublevels associated with different total spin orientations. In L–S coupling the spin–orbit interaction is treated as a small perturbation to the electrostatic Hamiltonian. The usual first-order perturbation Hamiltonian for spin–orbit coupling (effective form) is proportional to $\mathbf{L} \cdot \mathbf{S}$. Thus the first-order energy correction is proportional to the expectation value $\langle \mathbf{L} \cdot \mathbf{S} \rangle$.

Use the vector identity:

$$\mathbf{J}^2 = (\mathbf{L} + \mathbf{S})^2 = \mathbf{L}^2 + \mathbf{S}^2 + 2 \mathbf{L} \cdot \mathbf{S}.$$

Rearrange to express $\mathbf{L} \cdot \mathbf{S}$:

$$\mathbf{L} \cdot \mathbf{S} = \frac{1}{2} (\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2).$$

Replace operators by eigen values (for a state with quantum numbers L, S, J):

$$\langle \mathbf{L} \cdot \mathbf{S} \rangle = \frac{1}{2} [J(J + 1) - L(L + 1) - S(S + 1)]\hbar^2.$$

If the spin–orbit interaction is written as $H_{so} = \zeta \mathbf{L} \cdot \mathbf{S}$ (where ζ is an effective spin–orbit parameter depending on the configuration and radial integrals), the energy shift of the term $^{2S+1}L_J$ is:

$$\Delta E_J = \zeta \langle \mathbf{L} \cdot \mathbf{S} \rangle = \frac{\zeta \hbar^2}{2} [J(J + 1) - L(L + 1) - S(S + 1)].$$

Commonly the \hbar^2 factor is absorbed into ζ , so the frequently quoted form is

$$\boxed{\Delta E_J = \frac{\zeta}{2} [J(J + 1) - L(L + 1) - S(S + 1)]}.$$



This is the basic formula used to compute spin–orbit splitting of a given LS term.

From the energy formula above, consider two consecutive J-levels in the same term, J and $J - 1$. The energy difference is

$$\Delta E(J) - \Delta E(J - 1) = \frac{\zeta}{2} ([J(J + 1) - (J - 1)J])$$

since $L(L + 1)$ and $S(S + 1)$ cancel between the two. Compute the bracket:

Compute $J(J + 1) - (J - 1)J$ carefully:

- $J(J + 1) = J^2 + J$.
- $(J - 1)J = J^2 - J$.

Subtract: $(J^2 + J) - (J^2 - J) = 2J$.

Thus

$$\Delta E(J) - \Delta E(J - 1) = \frac{\zeta}{2} \cdot 2J = \zeta J.$$

Therefore the interval between consecutive J-levels is proportional to J . This is the

Landé interval rule:

$$\boxed{E(J) - E(J - 1) = \zeta J}.$$

This rule yields characteristic spacing ratios among the spin–orbit components of an LS term.

Take an LS term with $L = 1$ (P) and $S = 1$ (triplet). Allowed J values are $J = |L - S|, \dots, L + S \rightarrow J = 0, 1, 2$.

Apply the energy formula:

$$\Delta E_J = \frac{\zeta}{2} [J(J + 1) - L(L + 1) - S(S + 1)].$$



Compute the constants:

- $L(L + 1) = 1(1 + 1) = 1 \times 2 = 2.$
- $S(S + 1) = 1(1 + 1) = 2.$

$$\text{So } L(L + 1) + S(S + 1) = 4.$$

Now compute $J(J + 1)$ for each J and then ΔE_J :

1. For $J = 2$:

$$J(J + 1) = 2 \times 3 = 6.$$

$$\Delta E_2 = \frac{\zeta}{2}(6 - 4) = \frac{\zeta}{2} \cdot 2 = \zeta.$$

2. For $J = 1$:

$$J(J + 1) = 1 \times 2 = 2.$$

$$\Delta E_1 = \frac{\zeta}{2}(2 - 4) = \frac{\zeta}{2} \cdot (-2) = -\zeta.$$

3. For $J = 0$:

$$J(J + 1) = 0 \times 1 = 0.$$

$$\Delta E_0 = \frac{\zeta}{2}(0 - 4) = \frac{\zeta}{2} \cdot (-4) = -2\zeta.$$

So the three energies (relative) are ζ , $-\zeta$, -2ζ . The spacing:

- $E(2) - E(1) = \zeta - (-\zeta) = 2\zeta = \zeta \cdot 2$ which equals ζJ with $J = 2$.
- $E(1) - E(0) = -\zeta - (-2\zeta) = \zeta = \zeta \cdot 1$ which equals ζJ with $J = 1$.

This verifies the Landé interval rule.

Note on ordering: The absolute ordering (which J is highest/lowest) depends on the sign of ζ and on whether the shell is less than half-filled or more than half-filled (Hund's third rule). Typically:

- For **less than half-filled** shell, the level with **lowest** J is lowest in energy.



- For **more than half-filled** shell, the level with **highest J** is lowest in energy.

This together with the sign of ζ determines the observed ordering.

For a multi-electron configuration you obtain LS terms by:

1. Combine individual l_i to get allowed L values (vector addition and accounting for identical-electron antisymmetry).
2. Combine individual s_i to get allowed S values.
3. For each pair (L, S) list the term ^{2S+1}L and allowed J values.

Example: two equivalent p electrons (p^2 , each $l = 1$, each $s = 1/2$):

- Combine two $l = 1 \rightarrow$ allowed $L = 0, 1, 2$ (S, P, D).
- Combine two $s = 1/2 \rightarrow$ allowed $S = 0, 1$ (singlet, triplet).

(You can find these by constructing microstates and applying antisymmetry; for exam answers it is sufficient to state the allowed terms for common small configurations.)

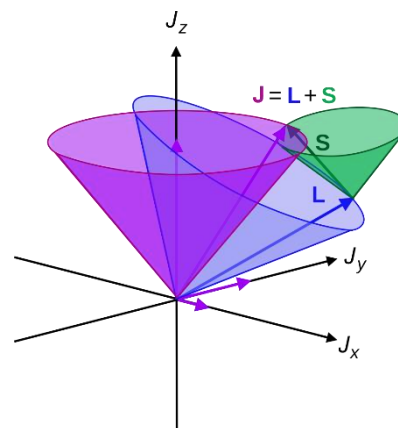


Figure 1. 2

Hund's rules (in context of LS coupling)

Hund's rules predict the ground term of an electron configuration (useful when writing term symbols before including spin-orbit splitting):

1. **Maximum multiplicity rule:** The term with maximum S (maximum multiplicity) lies lowest in energy.
2. **Maximum L rule:** For terms with same S , the term with largest L lies lowest.
3. **For less/greater than half-filled shells:** For subshell less than half-filled, the level with **lowest J** lies lowest; for more than half-filled, the level with **highest J** lies lowest.



Hund's rules are empirical but follow from electron repulsion and spin-orbit considerations and are consistent with LS coupling.

Limitations of L–S coupling

- Valid primarily for **light atoms**; for heavy atoms spin-orbit interaction becomes comparable to or larger than electrostatic interactions and **J–J coupling** (or intermediate coupling) becomes more appropriate.
- The simple perturbative formula uses a single effective ζ ; accurate numerical values require radial integrals and configuration interaction calculations.
- For equivalent electrons the construction of allowed terms requires careful anti symmetrization (Pauli principle).

J – J Coupling

J–J coupling is a type of angular momentum coupling in multi-electron atoms where spin-orbit interaction for each electron is stronger than electron–electron repulsion.

Therefore, each electron's orbital angular momentum (l_i) and spin angular momentum (s_i) couple within the same electron to form its individual total angular momentum j_i . Then all the j_i values combine to give the total angular momentum J of the atom. This coupling scheme is most important for heavy atoms (high Z) where spin-orbit coupling increases significantly.

Spin-orbit coupling increases roughly as:

$$\text{Spin-orbit strength} \propto Z^4$$

Hence, in heavy atoms:

- Spin-orbit interaction **within an electron** is very strong
- Electron–electron repulsion becomes comparatively weak



Thus, each electron behaves like an independent angular momentum system.

This leads to **J–J coupling**.

Step 1: Individual coupling within each electron

For each electron:

$$\mathbf{j}_i = \mathbf{l}_i + \mathbf{s}_i$$

Magnitude of j_i is quantised:

$$j_i = l_i + s_i, l_i + s_i - 1, \dots, |l_i - s_i|$$

Step 2: Total angular momentum of the atom

The total J is obtained by vector addition of all j_i :

$$\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2 + \mathbf{j}_3 + \dots$$

Magnitude of J is quantised:

$$J = j_1 + j_2 + \dots, j_1 + j_2 + \dots - 1, \dots, |j_1 - j_2 - \dots|$$

Derivation of Total Angular Momentum Energy

Each electron's spin–orbit interaction energy is:

$$E_{so} = \zeta(r) \mathbf{l}_i \cdot \mathbf{s}_i$$

Using:

$$\mathbf{l}_i \cdot \mathbf{s}_i = \frac{1}{2} [j_i(j_i + 1) - l_i(l_i + 1) - s_i(s_i + 1)]\hbar^2$$

Total spin–orbit energy of the atom:

$$E = \sum_i \frac{\zeta_i}{2} [j_i(j_i + 1) - l_i(l_i + 1) - s_i(s_i + 1)]\hbar^2$$

This energy splitting becomes significant when Z is large \rightarrow hence J–J coupling dominates.

Consider a heavy atom with two electrons:

- Electron 1: $l_1 = 1$ (p-electron), $s_1 = 1/2$
- Electron 2: $l_2 = 2$ (d-electron), $s_2 = 1/2$



Step 1: Find j_1 and j_2

For electron 1 (p, $l=1$):

$$j_1 = \frac{3}{2}, \frac{1}{2}$$

For electron 2 (d, $l=2$):

$$j_2 = \frac{5}{2}, \frac{3}{2}$$

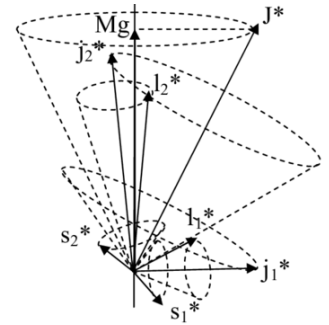
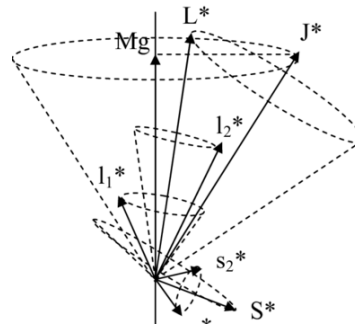


Figure 1. 3

Step 2: Possible total J values

If $j_1 = 3/2$ and $j_2 = 5/2$:

$$J = j_1 + j_2, j_1 + j_2 - 1, \dots, |j_1 - j_2|$$

$$J = 4, 3, 2, 1$$

So allowed J levels = **1, 2, 3, 4**.

Each J represents a separate energy level in the spectrum.

L-S coupling uses term symbols like:

$$^{2S+1}L_J$$

But in J-J coupling:

- We describe **each electron by n, l, j**
- Then combine the j-values

Example notation:

$$(3p_{1/2})(3d_{5/2})_J$$

or

$$j_1 j_2 J$$

This shows the independent coupling of electrons.

Transition selection rules:

(i) $\Delta J = 0, \pm 1$



But:

$$J = 0 \leftrightarrow j = 0$$

(ii) Parity must change

(Important for heavy atoms)

(iii) Individual j-values change according to spin-orbit allowed rules

These rules help explain fine structure in heavy atoms.

Where J–J Coupling is Important

- Heavy elements ($Z > 40$)
- Lanthanides and Actinides
- X-ray spectra of heavy atoms
- Fine structure multiplets in high-Z atoms
- Atomic transitions in astrophysics (heavy elements in stars)

Difference between J–J and L–S Coupling

Feature	L–S Coupling	J–J Coupling
Applies to	Light atoms	Heavy atoms
Dominant interaction	Electron–electron repulsion	Spin–orbit interaction
First combined	All L's \rightarrow L and all S's \rightarrow S	Each $l_i + s_i \rightarrow j_i$
Total angular momentum	$J = L + S$	$J = j_1 + j_2 + \dots$



Term symbol	$(^{2S+1}L_J)$	$((n l_j)(n' l'_{j'})_J)$
Spectra type	Multiplets	Complex fine structure

Pauli's Exclusion Principle

The Pauli Exclusion Principle was proposed by Wolfgang Pauli in 1925 to explain the structure of atomic spectra and electron distribution in atoms. It is one of the most fundamental principles of quantum mechanics and governs the behaviour of fermions (particles with half-integer spin).

Statement of the Pauli Exclusion Principle

“No two electrons in an atom can have the same set of four quantum numbers.”

The four quantum numbers are:

1. **Principal quantum number (n)**
2. **Azimuthal quantum number (l)**
3. **Magnetic quantum number (m_l)**
4. **Spin quantum number (m_s)**

This means that:

- Each electron in an atom occupies a **unique quantum state**.
- A maximum of **two electrons** can occupy an orbital, and **they must have opposite spins**.

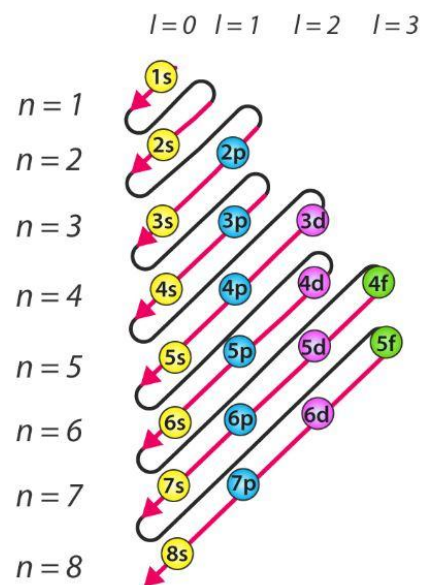


Figure 1. 4

Basis of the Principle (Quantum Explanation)

- Electrons are fermions ($\text{spin} = \frac{1}{2}$).

According to quantum mechanics, fermions obey anti-symmetric wavefunctions:

$$\Psi(1,2) = -\Psi(2,1)$$



If two electrons try to occupy the same quantum state:

$$\Psi(1,2) = \Psi(2,1)$$

Combining both:

$$\Psi = 0$$

A wavefunction equal to zero means the state cannot exist.

Thus, two identical electrons cannot be in the same state. This is the quantum mechanical derivation of Pauli's rule.

An orbital is defined by:

- n
- l
- m_l

Inside a single orbital, the allowed spin values are:

$$m_s = +\frac{1}{2}, -\frac{1}{2}$$

Therefore, **maximum electrons per orbital = 2**.

Two electrons in the same orbital must differ in **spin quantum number**.

Role in the Structure of the Periodic Table

Pauli's principle dictates:

- How electrons fill shells and subshells
- The electron configuration of every element
- The periodicity of chemical properties

Examples:

- $1s$ can hold 2 electrons



- 2p has three orbitals \rightarrow total 6 electrons
- 3d has five orbitals \rightarrow total 10 electrons

Thus, the **period lengths** of the periodic table directly arise from the maximum allowed electron occupancy.

Consequences of the Pauli Exclusion Principle

1. Stability of Matter

Matter does not collapse because electrons cannot all fall into the lowest energy level—each must occupy a separate quantum state.

2. Electron Configurations

Determines the arrangement of electrons (Aufbau principle is based on Pauli + energy order).

3. Degeneracy in Spectra

Energy levels split because electrons must have unique quantum numbers.

4. White Dwarf Stars (Astrophysics)

Electron degeneracy pressure, which supports white dwarfs from collapsing, originates from Pauli's exclusion.

Pauli Principle & Vector Atom Model

In the vector atom model:

- Each electron is described by quantum numbers (n, l, j, m_j)
- No two electrons in an atom can have all the same **n, l, j, m_j**

This restricts:

- Allowed electron arrangements
- Coupling schemes (L–S and J–J)



- Energy levels of multi-electron atoms

Quantum Number	Electron 1	Electron 2
n	2	2
l	1	1
m_l	0	0
m_s	$+\frac{1}{2}$	$-\frac{1}{2}$

Summary

- Pauli Exclusion Principle states that no two electrons in an atom can have identical sets of four quantum numbers.
- It explains the structure of electron shells, the periodic table, atomic spectra, and the stability of matter.
- It arises naturally from the anti-symmetric wavefunction of fermions.
- It limits the occupancy of an orbital to two electrons with opposite spins.

Magnetic Dipole moment due to orbital motion and spin motion of the electron

Electrons possess **orbital motion** and **spin motion**. Both motions produce **magnetic dipole moments** due to the motion of electric charge. Thus, an electron has two magnetic moments:

1. **Orbital magnetic dipole moment**
2. **Spin magnetic dipole moment**

Both are crucial to understand:

- Fine structure
- Zeeman effect
- Vector atom model
- Magnetic behaviour of atoms

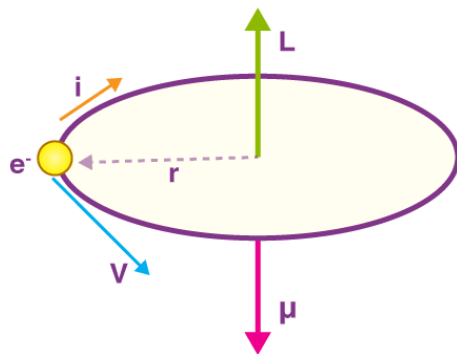


Figure 1. 5



- When an electron moves around the nucleus in an orbit, it behaves like a tiny current loop.
- A moving negative charge produces a magnetic field \rightarrow hence a magnetic dipole moment.

Current produced by orbital motion:

$$I = \frac{e}{T}$$

where

- e = electron charge
- T = time period

Orbital magnetic moment:

$$\mu_L = I \cdot A$$

Using Bohr's angular momentum:

$$L = mvr = n\hbar$$

After substituting and simplifying, the magnetic moment becomes:

$$\mu_L = \frac{e}{2m} L$$

The term $\frac{e}{2m}$ is called the gyromagnetic ratio.

Magnitude of orbital angular momentum:

$$L = \sqrt{l(l+1)}\hbar$$

Thus:

$$\mu_L = \frac{e\hbar}{2m} \sqrt{l(l+1)}$$

Since $\frac{e\hbar}{2m} = \mu_B$ (Bohr magneton),

$$\mu_L = \mu_B \sqrt{l(l+1)}$$



This is the orbital magnetic moment.

The electron also spins about its own axis.

Although spin is purely quantum mechanical, it behaves like a circulating charge → generating a spin magnetic moment.

Spin angular momentum:

$$S = \sqrt{s(s+1)}\hbar$$

For an electron, $s = \frac{1}{2}$.

Spin magnetic moment:

$$\mu_s = g_s \cdot \frac{e}{2m} S$$

where

- g_s = spin g-factor ≈ 2.0023

(Often taken as 2 in simple derivations)

Thus:

$$\mu_s \approx 2 \cdot \frac{e}{2m} S$$

$$\mu_s \approx \frac{e}{m} S$$

Using $S = \sqrt{s(s+1)}\hbar$:

$$\mu_s = g_s \mu_B \sqrt{s(s+1)}$$

For electron ($s = 1/2$):

$$\sqrt{s(s+1)} = \sqrt{\frac{1}{2} \left(\frac{3}{2}\right)} = \frac{\sqrt{3}}{2}$$

So:



$$\mu_S = g_S \mu_B \frac{\sqrt{3}}{2}$$

This is the spin magnetic moment.

If orbital (L) and spin (S) moments combine → total magnetic moment is linked to:

$$J = L + S$$

The total moment:

$$\mu_J = g_J \mu_B \sqrt{J(J+1)}$$

where g_J is the Lande g-factor used in vector atom model and Zeeman effect.

Summary

- Electron has two magnetic moments: orbital (μ_L) and spin (μ_S).
- Orbital magnetic moment:

$$\mu_L = \mu_B \sqrt{l(l+1)}$$

- Spin magnetic moment:

$$\mu_S = g_S \mu_B \sqrt{s(s+1)}$$

- Spin g-factor ≈ 2 , making spin magnetic moment almost twice the orbital moment.
- The combination of μ_L and μ_S determines Zeeman effect, fine structure, and energy splitting in atoms.
- Total magnetic moment of an atom is given by Lande's g-factor.

Bohr Magnetron

In atomic physics, the magnetic properties of electrons play a fundamental role in explaining phenomena such as the Zeeman effect, fine structure, and the magnetic



characteristics of atoms. The Bohr magneton, denoted by μ_B , is defined as the natural unit of magnetic dipole moment associated with the orbital motion of an electron in an atom. It represents the magnitude of the magnetic moment arising from the lowest permitted orbital angular momentum of an electron (i.e., $L = \hbar$). The concept of the Bohr magneton originates from the idea that the orbital motion of an electron about the nucleus is equivalent to an electric current loop. A moving charge generates a magnetic field, and consequently it possesses a **magnetic dipole moment**.

Consider an electron of charge $-e$, mass m_e , moving in a circular orbit of radius r with velocity v .

The time period of revolution is:

$$T = \frac{2\pi r}{v}$$

Therefore, the equivalent electric current is:

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

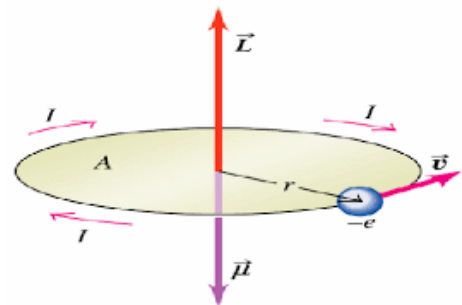


Figure 1. 6

The magnetic dipole moment of a circular loop is:

$$\mu = IA$$

where $A = \pi r^2$ is the area of the orbit.

Substituting the expressions for I and A :

$$\mu = \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{evr}{2}$$



The orbital angular momentum of the electron is:

$$L = m_e vr$$

Thus the magnetic moment becomes:

$$\mu = \frac{e}{2m_e} L$$

Therefore, the magnetic moment is **proportional** to the orbital angular momentum.

Using the quantum condition for the *minimum* orbital angular momentum:

$$L = \hbar$$

the smallest magnetic moment associated with orbital motion becomes:

$$\mu_B = \frac{e\hbar}{2m_e}$$

This quantity is known as the **Bohr magneton**.

Expression and Numerical Value

$$\mu_B = \frac{e\hbar}{2m_e}$$

Substituting standard constants:

- $e = 1.602 \times 10^{-19} \text{ C}$
- $\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$
- $m_e = 9.109 \times 10^{-31} \text{ kg}$

$$\mu_B = 9.274 \times 10^{-24} \text{ J/T}$$



In Gaussian units:

$$\mu_B = 0.9274 \times 10^{-20} \text{ erg/Gauss}$$

Physical Significance

1. Fundamental unit:

The Bohr magneton is the fundamental quantum of magnetic moment for electrons.

All orbital and spin magnetic moments are expressed in terms of μ_B .

2. Orbital magnetic moment:

For an electron in an orbital with quantum number l :

$$\mu_L = \mu_B \sqrt{l(l+1)}$$

3. Spin magnetic moment:

Because the electron spin g-factor $g_s \approx 2$:

$$\mu_S \approx g_s \mu_B \sqrt{s(s+1)}$$

4. Zeeman effect:

Energy splitting in a magnetic field is directly proportional to μ_B .

5. Atomic spectra:

μ_B determines the degree of splitting in fine structure levels through spin-orbit coupling.

The term $\frac{e}{2m_e}$ appearing in the definition of μ_B is the **orbital gyromagnetic ratio** (also known as the classical gyromagnetic ratio):

$$\gamma_L = \frac{e}{2m_e}$$

Thus:



$$\mu_B = \gamma_L \hbar$$

This shows that the Bohr magneton is the quantum of magnetic moment associated with **one unit of angular momentum** (\hbar).

In the vector atom model, the total magnetic moment of an electron arising from orbital and spin angular momenta is expressed in units of μ_B :

$$\mu_j = g_j \mu_B \sqrt{J(J+1)}$$

Thus, the Bohr magneton provides the **scale** for all magnetic effects in atomic structure.

Summary

- The Bohr magneton is the unit of magnetic dipole moment of an electron arising from its orbital motion.
- It is defined as $\mu_B = \frac{e\hbar}{2m_e}$.
- Numerical value: 9.274×10^{-24} J/T.
- Serves as a reference unit for both spin and orbital magnetic moments.
- Plays a central role in describing the Zeeman effect, fine structure splitting, and magnetic behaviour of atoms.
- Represents the magnetic moment corresponding to the minimum quantum of orbital angular momentum \hbar .

Stern-Gerlach Experiment

The Stern–Gerlach experiment, performed by Otto Stern and Walther Gerlach in 1922, is a foundational experiment in quantum mechanics. It demonstrated the quantization of angular momentum and provided direct evidence for the existence of electron spin.



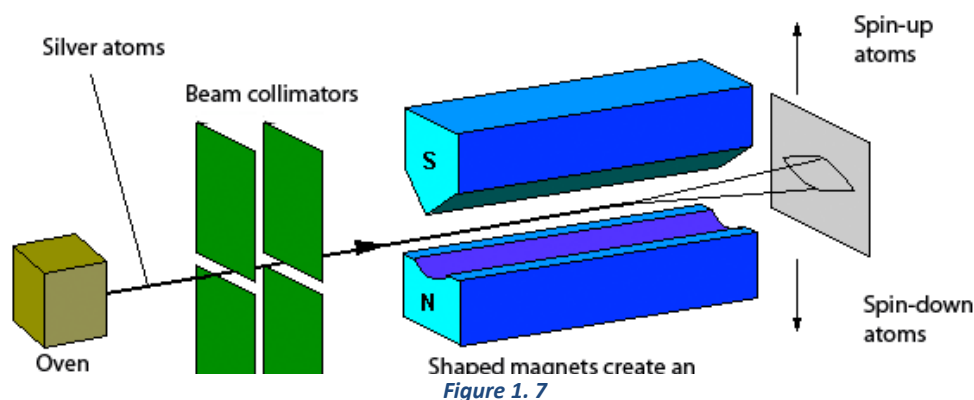
Objective of the Experiment

The purpose of the experiment was to study the behaviour of the magnetic dipole moment of atoms when passed through a non-uniform magnetic field. Classically, magnetic moments were expected to orient in arbitrary directions, producing a continuous spread of deflection. Quantum theory predicted discrete orientations, known as space quantization.

Experimental Setup

The apparatus consisted of the following components:

- (a) Source of atoms: Neutral silver atoms were generated by heating silver in a small oven. Silver was chosen because it has one unpaired electron in the 5s orbital, giving it a single resultant magnetic moment.
- (b) Collimated beam: The atoms emerging from the oven were passed through a narrow slit, forming a fine, collimated beam.
- (c) Non-Uniform Magnetic Field
 - Two magnets designed so that:
 - One pole piece is sharply curved (knife-edge)
 - The other is flat
 - This creates a strong magnetic field gradient ($\partial B/\partial z \neq 0$).
 - A magnetic dipole in this gradient experiences:





$$F = \mu_z \frac{\partial B}{\partial z}$$

(d) Detection screen: The deflected atoms were collected on a glass plate or photographic plate to observe the pattern.

Expected (Classical) Result

- Magnetic moment can orient in infinite possible angles.
- So forces on atoms vary continuously.
- The beam should spread into a continuous band on the screen but this did not occur.

Actual Result

The beam split into two distinct spots.

This proves:

1. Magnetic moment has only two possible orientations with respect to the magnetic field.
2. These correspond to:

$$m_s = +\frac{1}{2}, \frac{1}{2}$$

Hence, silver atoms behave as if their single unpaired electron has only two quantized spin states.

Major Conclusions

(i) Existence of Electron Spin

- The experiment gave the first direct evidence that electrons possess an intrinsic angular momentum.
- Spin produces its own magnetic moment.

(ii) Space Quantization



Magnetic moment vectors do not orient freely; instead they have discrete quantized orientations.

For spin:

$$\mu_z = m_s g_s \mu_B$$

Where:

- $m_s = \pm \frac{1}{2}$
- $g_s \approx 2$ (spin g-factor)
- μ_B = Bohr magneton

(iii) Quantum States Are Discrete

Particles don't take continuous values of angular momentum → they take allowed quantized states.

Total magnetic moment due to spin:

$$\vec{\mu}_s = -g_s \frac{e}{2m_e} \vec{S}$$

Component along magnetic field direction (z-axis):

$$\mu_{sz} = -g_s \frac{e}{2m_e} m_s \hbar$$

Force on the magnetic dipole:

$$F_z = \mu_{sz} \frac{\partial B}{\partial z}$$

Since $m_s = \pm \frac{1}{2}$:

Atoms get deflected upward or downward in two sharp beams.

Silver Atoms Were Used



Silver (Ag):

- Electronic configuration: $[\text{Kr}] 4d^{10} 5s^1$
- Only one unpaired 5s electron \rightarrow gives a net magnetic moment.
- Neutral atoms avoid complications from electric forces

Importance of Stern–Gerlach Experiment

- ✓ Proved space quantization
- ✓ Showed direct evidence for electron spin
- ✓ Demonstrated that angular momentum is quantized
- ✓ Helped build modern quantum mechanics
- ✓ Opened path for nuclear magnetic resonance (NMR), MRI, spintronics



Unit 2: Atomic Spectra

1. Introduction
2. Zeeman Effect
3. Quantum mechanical explanation of normal Zeeman effect
4. Anomalous Zeeman Effect
5. Quantum Mechanical Theory
6. Fine Structure of Sodium D-lines
7. Paschen-Back Effect
8. Stark effect

Introduction

Every atom contains electrons that occupy certain fixed energy levels around the nucleus. These energy levels are not continuous; instead, they are sharply defined and quantized. Because of this, electrons cannot have just any energy. They can exist only in specific energy states.

When an atom receives energy (from heat, electricity, or radiation), an electron in the atom may absorb that energy and jump from a lower energy level to a higher one. This process is called excitation. However, an electron cannot remain in the excited state for long. It eventually returns to a lower energy level. During this return, the atom releases the absorbed energy in the form of electromagnetic radiation.

This emitted or absorbed radiation has a certain wavelength that depends on the energy difference between the two levels involved in the transition. Since every element has its own



unique set of energy levels, the pattern of wavelengths it produces is also unique. This pattern of wavelengths is called the atomic spectrum of that element.

Atomic spectra are typically seen as a series of lines, each line representing a specific electronic transition. Because the energy levels in atoms are fixed, the wavelengths of the spectral lines are also fixed. Thus, atomic spectra act like fingerprints for elements and help in identifying them.

1. Spectral Line

A single wavelength (or frequency) of radiation emitted or absorbed during an electronic transition. Each line corresponds to a specific energy difference between two levels.

2. Line Spectrum

A spectrum that contains only discrete wavelengths rather than a continuous range.

Atoms produce line spectra because their energy levels are quantized.

3. Emission Spectrum

A set of bright lines on a dark background produced when atoms emit radiation as electrons return to lower energy levels.

4. Absorption Spectrum

A set of dark lines on a bright background produced when atoms absorb specific wavelengths as electrons move to higher energy levels.

5. Continuous Spectrum

A spectrum containing all wavelengths without interruption. It is produced by solids, liquids, or dense gases.

6. Energy Levels (E_n)



Fixed, quantized states in which electrons exist within an atom. Denoted as:

E_1, E_2, E_3, \dots

where E_1 is the ground state (lowest energy).

7. Ground State

The lowest stable energy level of an atom ($n = 1$ for hydrogen).

8. Excited State

Any energy level higher than the ground state. Electrons reach these states by absorbing energy.

9. Transition

The movement of an electron from one energy level to another.

- Upward transition = absorption
- Downward transition = emission

10. Wavelength (λ)

The distance between two consecutive wave peaks. Represented by the Greek letter lambda (λ). Usually measured in meters, nanometers, or angstroms.

11. Frequency (ν)

The number of wave cycles per second. Represented by the Greek letter nu (ν).

Measured in hertz (Hz).

12. Rydberg Formula

Used to calculate the wavelength of hydrogen spectral lines:

$$1/\lambda = R (1/n_1^2 - 1/n_2^2), n_2 > n_1$$

13. Rydberg Constant (R)



A fundamental constant in atomic physics used in the Rydberg formula. Its approximate value is:

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

14. Principal Quantum Number (n)

A number that indicates an electron's energy level:

$$n = 1, 2, 3, 4, \dots$$

15. Spectral Series

Groups of spectral lines corresponding to transitions ending at the same lower energy level.

Examples in hydrogen: Lyman, Balmer, Paschen, Brackett, Pfund.

16. Ionization Energy

The energy required to completely remove an electron from the atom ($n = \infty$).

17. Photon

A particle of light representing a quantum of electromagnetic radiation. Its energy is given by:

$$E = h\nu$$

where h is Planck's constant.

18. Wavenumber ($\tilde{\nu}$)

The reciprocal of wavelength:

$$\tilde{\nu} = 1/\lambda$$

Measured in cm^{-1} . Commonly used in spectroscopy.

19. Term Symbol

A notation representing the energy state of electrons, written as:

$$2S+1LJ$$



Where

- S = total spin quantum number
- L = orbital angular momentum (S, P, D, F...)
- J = total angular momentum

(Used heavily in advanced spectroscopy, term notation, and fine structure analysis.)

20. Fine Structure

Small, detailed splitting of spectral lines due to spin–orbit interaction in atoms.

Zeeman Effect

The Zeeman Effect refers to the splitting of a single spectral line into several closely spaced lines when the source of the spectrum is placed in a magnetic field. This phenomenon was discovered by Pieter Zeeman in 1896. In a normal situation (without a magnetic field), transitions between atomic energy levels produce a single wavelength, appearing as a single spectral line. However, when an external magnetic field is applied, the energy levels are altered due to the interaction between the magnetic field and the magnetic moment of electrons. As a result, each spectral line splits into multiple components. This splitting of spectral lines in the presence of a magnetic field is called the Zeeman Effect.

Why the Zeeman Effect Occurs

Electrons moving in an orbit around the nucleus behave like tiny current loops. A current loop has a magnetic moment. Because electrons possess both orbital angular momentum and spin angular momentum, they have an associated magnetic moment.

THE ZEEMAN EFFECT

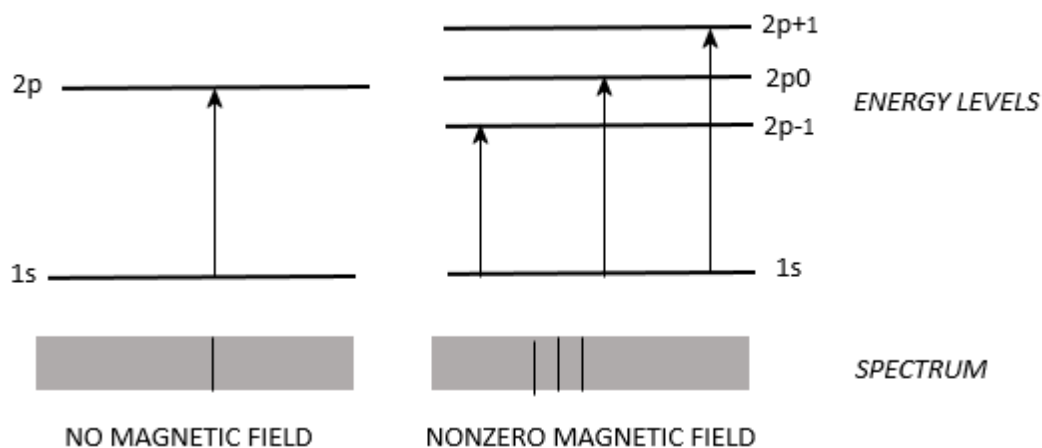


Figure 2. 1

When an external magnetic field is applied:

1. The magnetic moment interacts with the magnetic field.
2. The degeneracy (equal energy nature) of some energy levels is removed.
3. Energy levels split into multiple components depending on the value of the magnetic quantum number (m_l or m_J).
4. The transitions between these split energy levels produce additional spectral lines.

Thus, the Zeeman Effect is a direct result of the quantized magnetic properties of electrons.

In a magnetic field of strength B , an energy shift occurs given by:

$$\Delta E = m_l \mu_B B$$

or in the more general case (using total angular momentum J):

$$\Delta E = m_J g_J \mu_B B$$

Here,

μ_B = Bohr magneton

m_l = magnetic quantum number of orbital angular momentum

m_J = magnetic quantum number of total angular momentum

g_J = Lande g-factor



Because m_l or m_J can take multiple values, multiple energy shifts occur, causing multiple spectral lines.

1. Normal Zeeman Effect

The spectral line splits into three components:

- One unshifted line (central component)
- Two symmetrically shifted lines

This happens when the electron spin does not contribute to the magnetic moment (that is, spin quantum number is zero).

The splitting pattern is simple and can be explained using only orbital angular momentum.

Characteristics:

- Three components
- Observed in singlet states
- Explained by classical theory

2. Anomalous Zeeman Effect

The spectral line splits into more than three components.

This occurs when both orbital and spin magnetic moments interact with the magnetic field.

Most spectral lines in nature show anomalous Zeeman splitting.

Characteristics:

- Multiple components
- Observed in doublets, triplets, and complex spectra
- Requires quantum mechanics and Lande g-factor for explanation

Selection Rules for Transitions



Only transitions satisfying the following are allowed:

1. $\Delta m = 0$ (gives the central or π -component)
2. $\Delta m = \pm 1$ (gives the σ -components)

The π -component corresponds to radiation polarized parallel to the magnetic field.

The σ -components correspond to radiation polarized perpendicular to the magnetic field.

Experimental Observation

To observe the Zeeman Effect:

1. A light source (e.g., sodium vapor lamp) is placed between the poles of a strong electromagnet.
2. The magnetic field causes splitting of spectral lines.
3. A high-resolution spectrometer (e.g., Fabry–Perot interferometer) is used to resolve the split components.
4. The polarization of the lines can be studied using a polarizer.

Significance of the Zeeman Effect

1. Provides evidence for the magnetic properties of electrons.
2. Supports quantum theory, especially the concept of magnetic quantum numbers.
3. Helps measure magnetic field strength in laboratory and astrophysical environments.
4. Used in spectroscopy, atomic physics, and plasma diagnostics.
5. Assists in understanding fine-structure splitting and spin-orbit interaction.

The Zeeman Effect is the splitting of spectral lines in the presence of a magnetic field. It arises from the interaction of the magnetic field with the magnetic moment associated with orbital and spin motion of electrons. The normal effect gives three components, while the anomalous effect gives multiple components. Its study played a major role in establishing the quantum theory of atomic structure.



Quantum mechanical explanation of normal Zeeman effect

The normal Zeeman effect refers to the splitting of a spectral line into three distinct components when the emitting or absorbing atom is placed in an external magnetic field. In quantum mechanics, this effect occurs only when the total spin of the electrons involved is zero ($S = 0$). Because the electron spin makes no contribution, the effect is simpler than the anomalous Zeeman effect.

1. Conditions for the Normal Zeeman Effect

The normal Zeeman effect occurs when:

1. The electron configuration involved in the transition has
 $S = 0$ (singlet state).
2. The total angular momentum is purely orbital, so
 $J = L$.
3. Only orbital magnetic moments contribute to the interaction with the magnetic field.

Since $S = 0$, there is no spin-orbit coupling contribution, and the splitting depends only on the orbital angular momentum quantum number.

2. Magnetic Moment Due to Orbital Motion

In quantum mechanics, the orbital motion of an electron around the nucleus produces a magnetic moment given by:

$$\mu_L = - (e / 2m) L$$

The negative sign indicates that the electron's magnetic moment is opposite its angular momentum due to negative charge.

The magnitude of the orbital magnetic moment is expressed in terms of the Bohr magneton (μ_B):

$$\mu_B = e\hbar / 2m$$



3. Interaction of Magnetic Moment with External Magnetic Field

When an external magnetic field B is applied, the magnetic moment interacts with this field. The interaction energy is given by:

$$\Delta E = - \mu_L \cdot B$$

Because the magnetic field is usually applied along the z -axis, only the z -component of the orbital magnetic moment contributes:

$$\mu_L z = - m_l \mu_B$$

Here,

m_l = magnetic quantum number ($-\ell$ to $+\ell$)

Thus, the energy shift becomes:

$$\Delta E = m_l \mu_B B$$

This energy splitting is the origin of the normal Zeeman effect.

4. Splitting of Energy Levels

Without a magnetic field, all states with the same ℓ but different m_l have the same energy (they are degenerate).

When a magnetic field is applied:

$$\text{Energy of a state} = E_0 + m_l \mu_B B$$

Therefore, each level splits into $(2\ell + 1)$ equally spaced sub-levels.

For a transition between two states, the splitting in their energy differences produces multiple spectral components.

5. Selection Rules for Transitions

Quantum mechanical selection rules for electric dipole transitions are:

$$\Delta \ell = \pm 1$$

$$\Delta m_l = 0, \pm 1$$



Because $m\ell$ changes by these values, the spectral line splits into:

1. $\Delta m\ell = 0$ (π -component, unshifted)
2. $\Delta m\ell = +1$ (σ^+ component, shifted to higher frequency)
3. $\Delta m\ell = -1$ (σ^- component, shifted to lower frequency)

Thus, a single spectral line becomes three lines, producing the classical normal Zeeman triplet.

6. Frequency Shift

The change in energy for $\Delta m\ell = \pm 1$

is:

$$\Delta E = \pm \mu_B B$$

The corresponding shift in

frequency is:

$$\Delta \nu = \Delta E / h = \pm (\mu_B B / h)$$

Since the shifts are equal and

opposite, the spectrum shows symmetrical splitting.

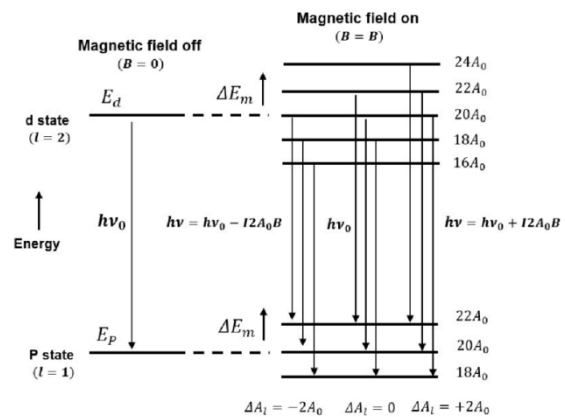


Figure 2. 2

7. Reason for Simplicity of the Normal Effect

In the normal Zeeman effect:

- $S = 0$
- $J = L$
- The Lande g-factor equals 1

Thus, the energy shift formula simplifies to:

$$\Delta E = mJ \mu_B B$$

with $mJ = m\ell$.



Because of this, only three transitions are possible, and the effect is fully explained by orbital angular momentum alone. No spin contribution is present, and no complicated fine structure arises.

The normal Zeeman effect is the quantum mechanically simple case of spectral line splitting in a magnetic field. It occurs in singlet states ($S = 0$). The splitting results from the interaction of the orbital magnetic moment with the magnetic field, leading to the removal of m_l degeneracy. Allowed transitions between these split sub-levels yield three spectral components, giving rise to the characteristic triplet pattern.

Anomalous Zeeman Effect

The anomalous Zeeman effect refers to the splitting of a spectral line into more than three components when an atom is placed in a magnetic field. Unlike the normal Zeeman effect, where only the orbital magnetic moment plays a role, the anomalous effect arises when both the orbital and spin magnetic moments of electrons interact with the external magnetic field. This effect is far more common in nature; most atoms and transitions show anomalous Zeeman splitting. Its complete explanation required quantum mechanics and the introduction of electron spin.

In many atoms, the total spin quantum number S is not zero.

Hence:

Total angular momentum $J = L + S$

Because both L (orbital angular momentum) and S (spin angular momentum) contribute magnetic moments, the interaction with the magnetic field becomes more complex.

As a result:



1. The energy levels split unevenly.
2. More than three spectral components appear.
3. The splitting depends on the total angular momentum J and the magnetic quantum number m_J .

Thus, the anomalous Zeeman effect directly demonstrates the role of electron spin in atomic structure.

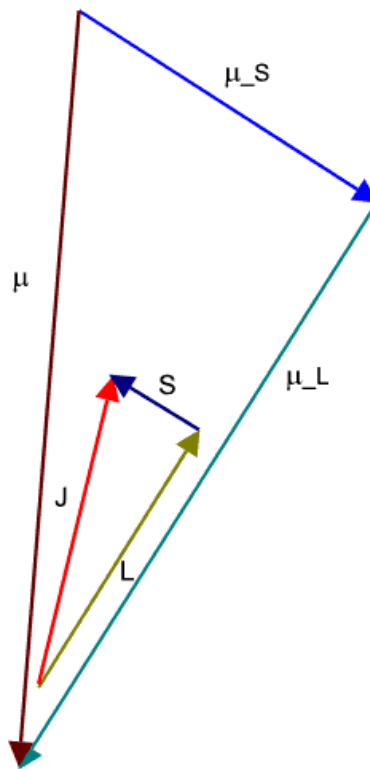


Figure 2. 3

Magnetic Moments Involved

Two magnetic moments contribute:

1. Orbital magnetic moment (μ_L)
Associated with electron's orbital motion.



2. Spin magnetic moment (μ_S)

Associated with intrinsic spin of the electron.

Both interact with the magnetic field.

Lande g-Factor

To account for the combined influence of L and S, the energy shift is described using the Lande g-factor:

$$g_J = 1 + [J(J + 1) + S(S + 1) - L(L + 1)] / [2J(J + 1)]$$

This factor determines how strongly a particular level interacts with the magnetic field.

In singlet states ($S = 0$), $g_J = 1$, giving the normal Zeeman effect.

In all other cases ($S \neq 0$), $g_J \neq 1$, producing the anomalous effect.

Energy Shift in a Magnetic Field

The shift in energy of a level is:

$$\Delta E = m_J g_J \mu_B B$$

where

m_J = magnetic quantum number ($-J$ to $+J$)

μ_B = Bohr magneton

B = external magnetic field

Because g_J and m_J take various values, the energy levels split into many unequally spaced sub-levels.

Splitting of Energy Levels

Without a magnetic field, all states with the same J but different m_J values have the same energy (degenerate).

When the field is applied:



$$\text{Energy} = E_0 + m_J g_J \mu_B B$$

Thus:

- A level with total angular momentum J splits into $(2J + 1)$ components.
- Different levels have different g_J values, so splitting is irregular.

The result is a complex pattern of lines.

Selection Rules for Transitions

Allowed transitions must satisfy:

$$\Delta m_J = 0, \pm 1$$

Thus, a transition between two levels that each split into many m_J sub-states results in multiple allowed transitions, producing several spectral lines.

No simple set of three lines arises, unlike the normal Zeeman effect.

Sodium D-Lines

The sodium D-lines (5890 Å and 5896 Å) are classic examples of anomalous Zeeman splitting.

Because the upper and lower levels involved have non-zero spin ($S \neq 0$), each splits into several components.

The transitions between these components produce a large number of spectral lines with unequal spacing. This provided experimental proof of electron spin.

Quantum Mechanical Importance

The anomalous Zeeman effect:

1. Confirmed the existence of electron spin.
2. Demonstrated the need for total angular momentum J .
3. Provided evidence for spin-orbit coupling.
4. Helped in developing the vector model of the atom.



5. Highlighted the concept of Lande g-factor.

Without the anomalous Zeeman effect, the understanding of atomic structure would have remained incomplete.

The anomalous Zeeman effect is the complex splitting of spectral lines in the presence of a magnetic field when $S \neq 0$. It arises from the combined interaction of orbital and spin magnetic moments with the magnetic field. The splitting pattern depends on J , m_J , and the Lande g-factor, leading to more than three spectral components. This effect is far more common than the normal Zeeman effect and played a major role in establishing modern quantum mechanics.

Quantum Mechanical Theory

The whole idea behind quantum mechanical atomic spectra is that **electrons inside an atom can only occupy fixed, quantized energy levels**, not continuous values. Because these levels are sharply defined, the energy changes during transitions are also sharply defined. These fixed energy differences directly produce the sharp spectral lines we observe.

Quantum mechanics explains exactly **why** these levels exist, **how** electrons behave in them, and **why only certain transitions are allowed**. According to quantum mechanics, an electron in an atom is described by a wavefunction. This wavefunction is a mathematical solution to the Schrödinger equation. When this equation is solved for the hydrogen atom, we get a set of discrete allowed energies. These energies depend mainly on the principal quantum number n . So only these fixed energy states are possible. There are no in-between energies. This automatically explains why atoms emit or absorb radiation only at certain wavelengths.

Every Energy Level Has Quantum Numbers

Each energy level of an electron is defined by four quantum numbers:



1. Principal quantum number n
2. Orbital angular momentum quantum number ℓ
3. Magnetic quantum number m_ℓ
4. Spin quantum number m_s

Quantum mechanics links these numbers to the energy, angular momentum, and orientation of the electron.

These quantum numbers also control which transitions are allowed or forbidden.

Spectral Lines Arise from Transitions

When an electron moves from a higher level (n_2) to a lower level (n_1), it emits a photon.

The energy of this photon is:

$$E = h\nu = E(n_2) - E(n_1)$$

Since $E(n_2)$ and $E(n_1)$ are fixed values, the frequency ν is also fixed.

So we get a sharp spectral line.

If the electron absorbs a photon and moves upward, absorption lines form.

Selection Rules

Not all transitions are allowed.

Quantum mechanics imposes rules called selection rules.

For hydrogen-like atoms, the major rules are:

$$\Delta\ell = \pm 1$$

$$\Delta m_\ell = 0, \pm 1$$



These rules arise from the properties of the dipole radiation operator.

Because of these restrictions, only certain jumps show up in the spectrum.

Fine Structure (Quantum Mechanical Correction)

Quantum mechanics also explains small corrections in energy levels due to:

- Spin–orbit coupling
- Relativistic mass correction
- Darwin term

These cause subtle splitting of spectral lines (fine structure), which classical models completely failed to explain.

Hyperfine Structure

Even smaller splittings occur due to the interaction between the electron's magnetic moment and the nuclear magnetic moment.

This gives hyperfine structure, also fully explained only by quantum mechanics.

Zeeman and Stark Effects

Quantum mechanics also explains how spectral lines split when:

- A magnetic field is applied (Zeeman effect)
- An electric field is applied (Stark effect)

These splittings arise from changes in energy depending on m_ℓ , m_s , and J values.

Hydrogen Spectrum and Rydberg Formula

The hydrogen spectrum fits beautifully with quantum mechanical energy levels:

$$E(n) = -13.6 \text{ eV} / n^2$$

This directly leads to the Rydberg formula.

So quantum mechanics gives a natural, mathematically derived explanation for all hydrogen spectral series (Lyman, Balmer, Paschen, etc.).

Multi-electron Atoms

For multi-electron atoms:

- Electron–electron repulsion
- Shielding
- Penetration
- Spin–orbit interactions



all modify the energy levels.

Quantum mechanics handles all these using approximate methods.

This explains complex spectra of heavier atoms.

Why Quantum Theory

Before quantum mechanics, classical theories failed to:

- Predict discrete lines
- Explain fine structure
- Explain anomalous Zeeman effect
- Explain intensity and polarization of lines
- Explain forbidden transitions
- Predict wavelengths accurately

Quantum mechanics explained all these consistently.

So the **entire structure of atomic spectra is a direct fingerprint of the quantum structure of atoms.**

Fine Structure of Sodium D-lines

The famous yellow glow of a sodium lamp is mainly due to two closely spaced spectral lines around 589 nm. These are known as the Sodium D-lines.

Earlier, it was believed to be a single line, but high-resolution spectroscopy showed that it is

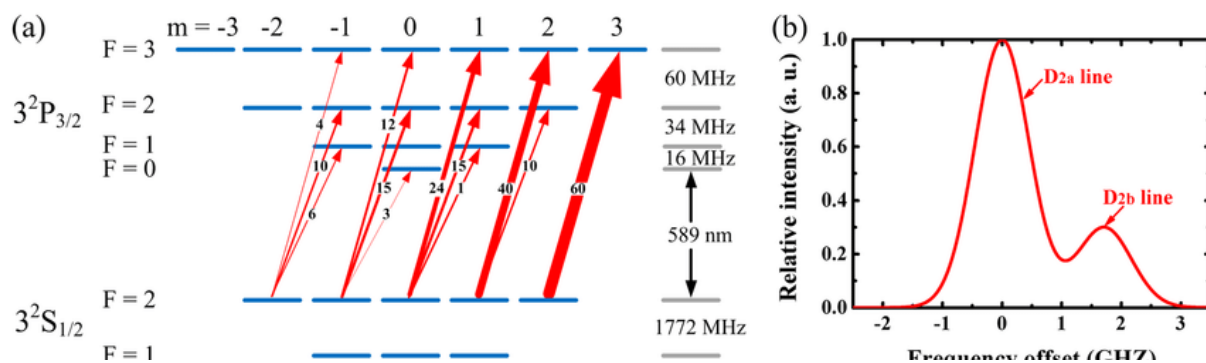


Figure 2.4



actually a pair of lines. This splitting is called **fine structure splitting**, and its origin is completely quantum mechanical.

The two lines are:

D1 line: 5896 \AA (or 589.6 nm)

D2 line: 5890 \AA (or 589.0 nm)

They appear extremely close because their energy difference is very small.

These lines arise from transitions between the **3p** and **3s** levels of the sodium atom.

Sodium: $1s^2 2s^2 2p^6 3s^1$

The valence electron moves from:

Excited state: 3p

Ground state: 3s

The 3s state has:

$$L = 0$$

$$S = 1/2$$

$$\text{So } J = 1/2$$

The 3p state has:

$$L = 1$$

$$S = 1/2$$

So J can be either:

$$J = 1/2$$

$$J = 3/2$$

Because of spin-orbit coupling, the 3p level splits into two separate energy levels:



$$3p\ ^2P_{1/2}$$

$$3p\ ^2P_{3/2}$$

This is the fine structure splitting that gives rise to the doublet lines.

Fine structure in sodium comes from:

1. **Spin–orbital interaction**

The electron's orbital motion creates a magnetic field.

Its spin magnetic moment interacts with this field.

2. **Relativistic correction**

Electrons moving closer to the nucleus behave relativistically.

This slightly affects their energy.

3. **Darwin term**

A quantum correction that applies mainly for s-states.

For the D-line case, the major cause is spin–orbit coupling.

The transitions are:

$$3p\ ^2P_{1/2} \rightarrow 3s\ ^2S_{1/2} \text{ (gives D1 line: } 5896\text{ \AA)}$$

$$3p\ ^2P_{3/2} \rightarrow 3s\ ^2S_{1/2} \text{ (gives D2 line: } 5890\text{ \AA)}$$

The upper-level splits into two because $J = 1/2$ and $J = 3/2$.

The lower state $3s\ ^2S_{1/2}$ has no orbital angular momentum ($L = 0$), so it does not split.

Thus, two different transitions are allowed:

$$^2P_{1/2} \rightarrow ^2S_{1/2}$$

$$^2P_{3/2} \rightarrow ^2S_{1/2}$$

These produce two lines.

The energy splitting between the $J = 1/2$ and $J = 3/2$ sublevels in sodium is small.

This produces wavelengths that differ by only 6 \AA (0.6 nm).



With ordinary spectrometers, they appear as a single line.

With high-resolution instruments, they appear as a doublet.

Significance of Sodium D-line Fine Structure

1. Strong experimental evidence for electron spin

The doublet cannot be explained without including spin.

2. Proof of spin–orbit coupling

Energy levels depend on J .

3. Support for the vector atom model

Shows L and S couple to form J .

4. Important in astrophysics

Used to study interstellar and solar atmospheres.

5. Classic example of fine-structure in alkali atoms

Sodium is the simplest multi-electron atom to show this clearly.

The sodium D-lines are a pair of yellow spectral lines at 5896 \AA and 5890 \AA . They originate from transitions between the $3p$ and $3s$ states. The $3p$ state splits into two levels ($J = 1/2$ and $J = 3/2$) due to spin–orbit coupling. The transitions from these two levels to the $3s$ level give rise to the D1 and D2 lines. This splitting is known as the fine structure.

Paschen-Back Effect

When an atom is placed in a magnetic field, its spectral lines split due to the interaction between the magnetic field and the magnetic moments associated with the orbital and spin angular momenta of electrons. For *weak* magnetic fields, this splitting is described by the **Zeeman effect**. However, when the magnetic field becomes *very strong*, the normal rules of the Zeeman effect no longer hold. The coupling between the orbital angular momentum L and

spin angular momentum \mathbf{S} breaks down. This breakdown leads to a different pattern of spectral splitting known as the **Paschen–Back Effect**.

This effect was discovered by Paschen and Back in 1921.

Paschen–Back Effect

In weak magnetic fields:

- \mathbf{L} and \mathbf{S} are coupled through **spin–orbit coupling**
- They combine to form total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$
- Zeeman splitting is calculated using \mathbf{J} and its magnetic quantum number m_J

But in very strong magnetic fields:

- The external magnetic field becomes stronger than the internal spin-orbit interaction.
- So \mathbf{L} and \mathbf{S} **no longer remain coupled**.
- Instead, they interact *independently* with the magnetic field.

This regime is where the **Paschen–Back Effect** appears.

Weak Field (Zeeman Region)

Spin–orbit coupling dominates:

\mathbf{L} and \mathbf{S} are strongly coupled $\rightarrow \mathbf{J}$ is a good quantum number.

Strong Field (Paschen–Back Region)

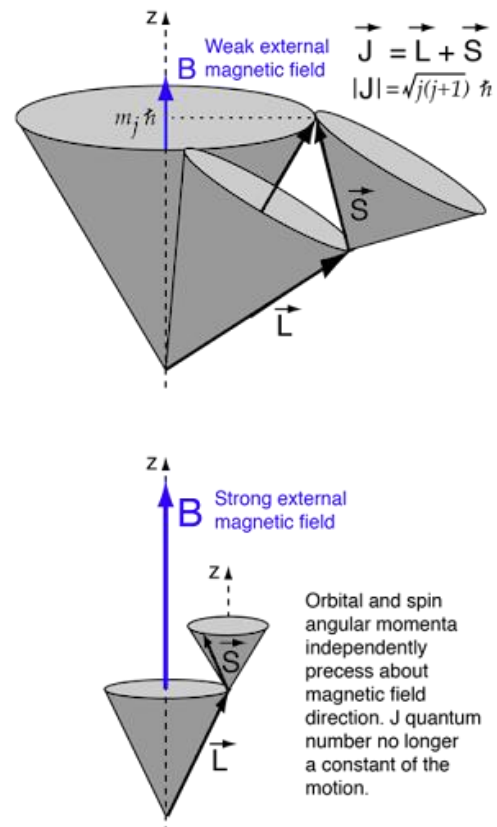


Figure 2.5



External magnetic field dominates:

L and S decouple $\rightarrow m_L$ and m_S become the good quantum numbers.

So in the Paschen–Back region:

- The spectral line shifts depend separately on m_L and m_S
- The splitting becomes more regular and symmetric
- The energy levels follow a simple linear pattern

Mathematical Treatment (Conceptual)

In Zeeman effect (weak field), energy shift is:

$$\Delta E = \mu_B g_J m_J B$$

But in the strong-field Paschen–Back case:

- J is no longer meaningful
- The magnetic field interacts separately with L and S

So the energy shift becomes:

$$\Delta E = \mu_B (m_L + g_S m_S) B$$

where:

- m_L is the orbital magnetic quantum number
- m_S is the spin magnetic quantum number
- $g_S \approx 2$ is the spin g-factor

This shows that the splitting pattern is now governed directly by m_L and m_S .

Characteristics of Paschen–Back Effect



1. **Occurs in very strong magnetic fields**
2. **Breakdown of spin–orbit coupling**
3. **m_L and m_S are good quantum numbers**
4. **Simpler and more symmetric splitting pattern**
5. **Transition from anomalous Zeeman effect to normal Zeeman-like pattern**
6. **Energy level splitting becomes linear in magnetic field strength**

Magnetic Field Strength	Effect Type	Coupling State
Weak	Normal / Anomalous Zeeman effect	L–S coupled \rightarrow J good
Intermediate	Transition region	Partial decoupling
Strong	Paschen–Back effect	L and S decoupled

In weak fields \rightarrow D-lines show **fine structure + anomalous Zeeman splitting**

In strong fields \rightarrow lines show **simple equal spacing** characteristic of Paschen–Back effect.

Importance

- Helps understand atomic structure under extreme magnetic environments
- Used in astrophysics to study strong magnetic fields of sunspots
- Provides experimental verification for electron spin and spin–orbit coupling
- Bridges weak-field Zeeman physics and full L-S decoupling physics

Stark Effect

The **Stark Effect** refers to the **splitting or shifting of atomic spectral lines when an atom is placed in an external electric field**. It is the electric-field analogue of the Zeeman effect



(which occurs in an external magnetic field). This effect was discovered by Johannes Stark in 1913. Inside an atom, electrons occupy quantized energy levels. These energies depend on:

- the attraction between the nucleus and electrons
- the electron's orbital motion
- interactions like spin-orbit coupling

When you apply an **external electric field**, it distorts the electron cloud around the nucleus. Because electric charge responds to electric fields, this distortion modifies the allowed energy levels of the electrons.

As a result:

- Some energy levels are **shifted**
- Some are **split into multiple sub-levels**

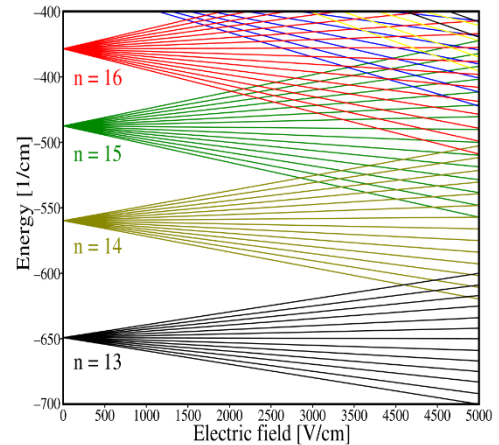


Figure 2. 6

Each transition between levels corresponds to a spectral line. So, when the levels shift or split, the spectral lines also shift or split.

This phenomenon is the **Stark Effect**.

1. Linear Stark Effect

Observed mainly in **hydrogen and hydrogen-like atoms** (especially in excited states).

Here, energy shifts are directly proportional to the electric field:

$$\Delta E \propto E$$

Reasons:

- Hydrogen has nearly degenerate energy levels
- Electric field lifts this degeneracy linearly

2. Quadratic Stark Effect



Observed in **most multi-electron atoms**.

Here, the shift is proportional to the square of the field:

$$\Delta E \propto E^2$$

Reasons:

- No degeneracy in normal multi-electron energy levels
- Response occurs through induced dipole moments \rightarrow quadratic dependence

In the presence of an electric field:

- The positive nucleus and negative electrons experience opposite forces
- This slightly **polarizes** the atom
- The atomic dipole moment changes
- Energy levels shift according to the interaction:

$$\Delta E = -\vec{p} \cdot \vec{E}$$

where

\vec{p} = electric dipole moment,

\vec{E} = external electric field.

In atoms without permanent dipole moments, the field **induces** a dipole moment, leading to quadratic shifts.

Quantum Mechanical Interpretation

Quantum mechanics explains the Stark Effect via **perturbation theory**.

The electric field adds a new term to the Hamiltonian:

$$H' = eEz$$



where z is the electron position along the field direction.

Depending on the symmetry and degeneracy of the initial energy levels:

- First-order perturbation gives **linear Stark effect**
- Second-order perturbation gives **quadratic Stark effect**

Observation in Spectra

In spectroscopy:

- Spectral lines **split** into multiple components
- Or they **shift** toward higher or lower wavelengths
- Patterns depend on the strength of the electric field
- Higher electric fields produce larger splittings

These shifts are used to measure electric fields in plasmas and astrophysical environments.

Aspect	Stark Effect	Zeeman Effect
External field	Electric	Magnetic
Causes	Distortion of electron cloud (electric dipole changes)	Interaction with magnetic dipoles
Splitting	Linear or quadratic	Normal, anomalous, or Paschen–Back
Typical use	Measuring electric fields	Measuring magnetic fields

Applications

1. **Spectral analysis of plasmas**
2. **Astrophysics** – measuring electric fields in stars and nebulae
3. **Laser physics** – Stark tuning of energy levels
4. **Microwave spectroscopy**



5. **Quantum computing** – controlling atomic states with electric fields
6. **Atomic clocks** – Stark shifts must be corrected for accuracy



Unit 3: Structure of Nuclei

1. Nuclear properties
2. Mass defect and Binding energy
3. Binding energy curve
4. Nuclear force (Characterstics)
5. Nuclear Models - Liquid drop
6. Mass formula Shell Model
7. Magic numbers
8. Evidences that led to shell

Nuclear properties

Atoms have a tiny, dense central core called the **nucleus**, which contains **protons** and **neutrons** (collectively called nucleons). Even though the nucleus is incredibly small compared to the whole atom, it holds almost all of the atom's mass. Understanding its basic properties helps explain nuclear stability, radioactivity, nuclear reactions, and more.

1. Nuclear Size

The nucleus is extremely small — its radius is about:

$$R = R_0 A^{1/3}$$

where,

- $R_0 \approx 1.2 \times 10^{-15} \text{ m}$
- A is the mass number



This equation shows that the nucleus gets larger as the number of nucleons increases, but not linearly — more like volume packing.

2. Nuclear Mass

The mass of a nucleus is approximately:

$$M = Zm_p + (A - Z)m_n - \text{Binding Mass}$$

- Z = number of protons
- $A - Z$ = number of neutrons

Nuclear masses are *slightly* less than the sum of individual proton + neutron masses due to **mass defect**, which is related to the binding energy.

3. Density of Nucleus

Nuclear density is **almost constant** for all nuclei.

$$\rho \approx 2.3 \times 10^{17} \text{ kg/m}^3$$

This insane density comes because the nucleus is packed tightly with nucleons.

Every nucleus — from hydrogen to uranium — has roughly the same density.

4. Nuclear Charge

The charge of the nucleus is only due to **protons**.

$$\text{Nuclear charge} = +Ze$$

- Z = number of protons
- e = electronic charge

So the identity of the element depends purely on its nuclear charge.



5. Nuclear Spin

Nuclei have **intrinsic angular momentum**, called **nuclear spin**.

It depends on:

- whether the number of protons is even/odd
- whether the number of neutrons is even/odd

Rules (quick version):

- Even Z + even $N \rightarrow \text{spin} = 0$
- Odd nucleon count \rightarrow non-zero spin

Spin influences:

- hyperfine structure
- NMR
- MRI
- atomic spectra

6. Nuclear Magnetic Moment

Since protons and neutrons have spin and charge (protons), many nuclei behave like tiny magnets.

The magnetic moment explains:

- Hyperfine splitting
- Zeeman interaction for nuclei
- NMR spectroscopy

Neutrons have magnetic moment too (despite having no charge) because they're internally made of quarks.

7. Nuclear Binding Energy



This is the energy that “locks” nucleons together.

$$E_b = \Delta mc^2$$

Higher binding energy → more stable nucleus.

The curve of binding energy explains:

- why iron is most stable
- why fusion works for light nuclei
- why fission works for heavy nuclei

8. Nuclear Stability

Stability depends on:

- neutron-to-proton ratio (N/Z)
- pairing effects
- shell effects
- binding energy per nucleon

Stable nuclei lie on the **stability belt** in the N–Z plot.

Unstable ones undergo:

- alpha decay
- beta decay
- gamma emission
- electron capture

9. Nuclear Shape

Nuclei are not always spherical.

They can be:

- spherical



- oblate (flattened)
- prolate (cigar-shaped)

Shape depends on spin, shell structure, and energy levels.

10. Nuclear Forces

Nucleons are held together by the **strong nuclear force**:

- short range ($\sim 1-3$ fm)
- attractive
- stronger than electrostatic repulsion
- saturating in nature

It overcomes proton–proton repulsion.

11. Nuclear Density

Okay so, nuclear density is literally one of the wildest things in physics.

Even though nuclei differ in size and number of nucleons, their **density is basically constant** for all elements.

Formula

The radius of a nucleus is:

$$R = R_0 A^{1/3}, R_0 \approx 1.2 \times 10^{-15} \text{ m}$$

The volume becomes:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

Mass is roughly Am_n (since neutron and proton masses \approx same).

So density:

$$\rho = \frac{Am_n}{\frac{4}{3}\pi R_0^3 A} = \frac{m_n}{\frac{4}{3}\pi R_0^3}$$

Notice the **A cancels out** — which is why **all nuclei have the same density**.

Value

$$\rho \approx 2.3 \times 10^{17} \text{ kg/m}^3$$

This is insanely high — like “if you compress a mountain into a spoon” type vibe.

Same order as **neutron-star density**.

Why all nuclei have same density



Because nucleons are basically packed in a saturated strong-force environment.

Strong nuclear force has:

- short range
- saturation (each nucleon interacts with only a few neighbours)

So adding more nucleons just adds more layers — the density stays constant.

12. Nuclear Parity

Now onto **parity**, which is more of a quantum-mechanics personality trait for nuclei.

Parity is basically the behavior of the **nuclear wavefunction** when you do a **space inversion**:

$$\vec{r} \rightarrow -\vec{r}$$

If the wavefunction stays the same:

$$\psi(-\vec{r}) = +\psi(\vec{r}) \Rightarrow \text{Positive Parity}$$

If it changes sign:

$$\psi(-\vec{r}) = -\psi(\vec{r}) \Rightarrow \text{Negative Parity}$$

Origin of Parity

Parity of a nucleus is determined by the **orbital angular momentum (l)** of the **unpaired nucleons**.

Formula:

$$\pi = (-1)^l$$

- If **l** is even (0, 2, 4...) \rightarrow parity = +
- If **l** is odd (1, 3, 5...) \rightarrow parity = -

Quick rules

- Nuclei with **even-even** nucleon number \rightarrow usually **positive parity** (ground state).
- Nuclei with **odd A** \rightarrow parity depends on the single unpaired nucleon.
- Parity doesn't care about spin directly — only about orbital angular momentum.

Why parity matters

Parity helps classify nuclear energy levels and transitions.

Selection rules for gamma transitions involve parity, like:

- Electric dipole (E1): changes parity
- Magnetic dipole (M1): does not change parity



So parity gives us the "yes/no" rules for allowed nuclear transitions.

Mass defect and Binding energy

If you add up the masses of *individual* protons and neutrons, that total is **always** a little more than the actual mass of the nucleus. This missing mass is called the **mass defect**.

Definition

$$\Delta m = [Zm_p + (A - Z)m_n] - M_{\text{nucleus}}$$

Where:

- Z = number of protons
- A = mass number
- m_p = mass of proton
- m_n = mass of neutron
- M_{nucleus} = actual measured nuclear mass

So, the nucleus weighs *less* than the sum of its nucleons.

Why is mass missing?

Because that mass has been converted into **energy** that acts like nuclear glue.

This energy is what binds the nucleus together.

Basically, the nucleus “paid” some mass to become stable.

Binding Energy

Binding energy is the **energy required to break the nucleus into its individual protons and neutrons**.

Or flipped around:

It's the **energy released when a nucleus is formed from free nucleons**.

This energy comes from the mass defect.

Formula

$$E_b = \Delta mc^2$$

Since c^2 is huge, even tiny mass losses give massive energies.

Binding Energy per Nucleon

This is the real measure of nuclear stability:



$$\frac{E_b}{A}$$

Higher value → nucleus is more tightly bound → more stable.

Typical Numbers

- Light nuclei: lower binding energy per nucleon
- Iron-56: **peak stability** (~8.8 MeV per nucleon)
- Very heavy nuclei (U-235): lower again → can undergo fission

This curve explains why:

- Light nuclei **fuse** (fusion releases energy)
- Heavy nuclei **split** (fission releases energy)

Why Mass Defect Happen

When nucleons bind:

- strong nuclear force pulls them together
- they move into a lower-energy configuration
- the “excess energy” is released
- that energy release shows up as a loss of mass

So, stable nuclei literally have **less mass than their ingredients** because energy has left the system.

Take deuterium (${}^1\text{H}^2$):

- 1 proton + 1 neutron should weigh a bit more
- actual deuteron mass is lower
- mass difference $\times c^2$ = binding energy (~2.2 MeV)

This energy is what you must supply to tear proton and neutron apart.

Concept	Meaning	Formula	Why it Matters
Mass Defect	Missing mass when nucleons form nucleus	$(\Delta m = (Zm_p + (A-Z)m_n) - M)$	Shows mass was converted to energy
Binding Energy	Energy needed to separate nucleus / released when formed	$(E_b = \Delta mc^2)$	Measures nuclear stability



Binding Energy per Nucleon	Stability indicator	(E _b /A)	Explains fusion & fission
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3.2 Binding energy curve

Every atom has a nucleus made of **protons** and **neutrons**, called **nucleons**. Individually, these nucleons have certain masses.

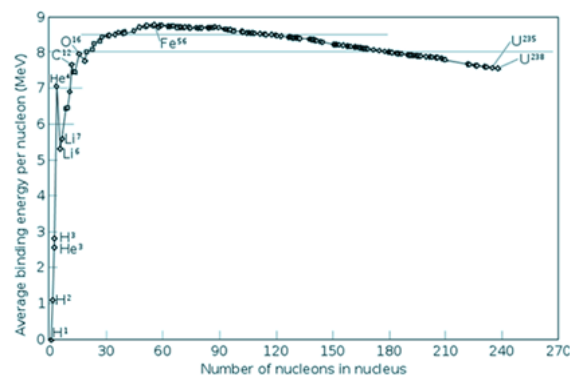


Figure 3. 1

But when they come together to form a nucleus, the nucleus weighs **less** than the sum of their individual masses. That “lost mass” turns into **energy** that binds the nucleus together. This is called **binding energy**. The **Binding Energy Curve** is basically a graph that shows how tightly bound different nuclei are. *Binding energy is the energy required to pull the nucleus apart into free protons and neutrons.*

Or in reverse:

It is the **energy released when a nucleus is formed** from free nucleons.

Formula

$$E_b = \Delta mc^2$$



Where

- Δm = mass defect
- c = speed of light

So even a tiny mass loss \rightarrow massive energy.

Binding Energy per Nucleon (The Real Star of the Topic)

To compare different nuclei, we don't look at total binding energy.

We look at **binding energy per nucleon**:

$$\frac{E_b}{A}$$

Higher value \rightarrow stronger binding \rightarrow more stable nucleus.

The Binding Energy Curve is literally a plot of this quantity versus mass number A .

Shape of the Binding Energy Curve

Let's go through the curve from left to right.

(A) Very light nuclei ($A = 1$ to ≈ 20)

- Binding energy per nucleon is low.
- As A increases, the value rises steeply.
- Why? Because more nucleons \rightarrow more strong-force attraction \rightarrow more stability.

Example:

Hydrogen (H), helium (He), carbon (C)

These can **fuse**, releasing energy, because the products have higher binding energy per nucleon.

(B) Middle-mass nuclei ($A \approx 40$ to 60)

This is where the curve reaches its **peak**.

Iron-56 and Nickel-62 have the highest binding energy per nucleon (~ 8.8 MeV).

This region contains the **most stable nuclei in the universe**.

You cannot get energy from splitting them, and you cannot get energy from fusing them — they're already at the top.

(C) Heavy nuclei ($A > 100$)

After iron, the curve gradually slopes downward.

Why?

- Too many protons \rightarrow too much repulsion
- Strong force saturates \rightarrow cannot hold everyone equally



- Nucleus becomes less stable

Heavy nuclei like Uranium-235 have **lower binding energy per nucleon**.

These can **undergo fission**, producing fragments with higher binding energy per nucleon → energy release.

Why Does the Curve Look Like This? (Physics Behind It)

Three major reasons:

(1) Strong Nuclear Force

- Acts only at short distances ($\approx 1-3$ fm)
- Very strong
- But *saturates* → each nucleon can only bond with a few neighbours

(2) Coulomb Repulsion

- Protons repel each other
- More protons = more repulsion
- Heavy nuclei become less stable

(3) Asymmetry of Neutrons and Protons

Too many neutrons or too many protons disturbs the balance → reduces stability.

These factors dominate at different ranges of A , giving the curve its shape.

Fusion (Left side of the curve)

Light nuclei have low BE/A .

When they fuse → product has higher BE/A → **energy released**.

This is how the sun produces energy.

Fission (Right side of the curve)

Heavy nuclei have lower BE/A .

When they split → fragments have higher BE/A → **energy released**.

This powers nuclear reactors.

Why Iron is the Most Stable

Iron-56 is at the peak.

You cannot extract energy by splitting or fusing it.

It is the "energy valley bottom" for nuclear processes.

Semi-Empirical Explanation

The shape of the binding energy curve is reproduced by the **Semi-Empirical Mass Formula**, which includes:



- volume term (strong force)
- surface term
- Coulomb term
- asymmetry term
- pairing term

These terms mathematically generate the same curve you see experimentally.

Summary

- Binding energy holds the nucleus together.
- Binding energy per nucleon measures stability.
- Light nuclei \rightarrow gain stability by fusion.
- Heavy nuclei \rightarrow gain stability by fission.
- Iron-56 is the most stable nucleus.
- The curve explains where nuclear energy comes from.
-

Nuclear force (Characteristics)

The nuclear force (often called the nucleon–nucleon force) is the fundamental interaction responsible for holding protons and neutrons together inside the atomic nucleus. Without this force, the electrostatic repulsion between positively charged protons would break the nucleus apart. Nuclear force overcomes this repulsion and stabilizes the nucleus. Protons and neutrons (collectively known as nucleons) are packed closely within a very small region of space, typically within a radius of about 1 femtometer ($1 \text{ fm} = 10^{-15} \text{ m}$). At such small distances, the repulsive Coulomb force between protons is extremely strong. The fact that nuclei remain bound indicates the existence of a strong, attractive non-electromagnetic force acting between nucleons. This force is the nuclear force.

Characteristics of Nuclear Force

1. Very strong at short range

The nuclear force is one of the strongest known forces but acts only within a very short distance (approximately 1–2 fm). Beyond this range, the force rapidly decreases to nearly zero.

Hence, it is a short-range force.



2. Attractive at medium range and repulsive at very short range
 - At distances around 0.8–2 fm, the force is strongly attractive.
 - Below about 0.5 fm, the force becomes repulsive.

This repulsive core prevents nucleons from collapsing into each other.
3. Charge independent

The force between two protons, two neutrons, or a proton and a neutron is approximately the same.

Therefore, nuclear force is essentially independent of electric charge.
4. Saturation property

Each nucleon interacts strongly only with its nearest neighbours.

This explains why nuclear density remains approximately constant and why binding energy per nucleon does not increase indefinitely with mass number.
5. Non-central force and dependent on spin orientation

Nuclear force does not depend only on the distance between nucleons but also on their relative orientation and spin states.

This gives rise to phenomena such as tensor force components.
6. Exchange force

Nuclear force arises due to the exchange of virtual mesons (Yukawa theory).

The exchange of particles produces an effective force between nucleons.
7. Non-conservative nature (in two-nucleon system)

It depends on the spins and is not strictly central, making it slightly different from classical conservative forces.

Origin of Nuclear Force (Yukawa Theory — brief)

Hideki Yukawa proposed that nuclear force originates due to the exchange of virtual mesons (especially pions) between nucleons.

This exchange mechanism naturally leads to a strong but short-range force, matching experimental observations.

Importance of Nuclear Force

1. It holds the atomic nucleus together.
2. It determines nuclear stability and the binding energy of nuclei.
3. It governs nuclear reactions, fission, and fusion.
4. It explains nuclear structure and energy levels.

Summary

Nuclear force is a strong, short-range, charge-independent, saturating force that binds nucleons together and stabilizes the nucleus. It has both attractive and repulsive regions and originates from the exchange of mesons. Understanding nuclear force is fundamental to nuclear physics, nuclear structure, and nuclear energy processes.

3.1 Nuclear Models – Liquid Drop

A nuclear model is a theoretical framework used to describe the structure, behaviour, and properties of the atomic nucleus. Because the nucleus is a complex system of interacting protons and neutrons, no single model can explain all its features. Therefore, different models are used to explain different nuclear phenomena.

Nuclear models attempt to describe:

- stability of nuclei,
- binding energy trends,
- nuclear spin and parity,
- magnetic moments,
- nuclear reactions,
- energy levels and spectra.

The main nuclear models are the liquid drop model, shell model, and collective model.

Liquid Drop Model

The liquid drop model is one of the earliest and most influential models of nuclear structure. It was developed to explain the average or bulk properties of nuclei, especially binding energy, nuclear stability, and the phenomenon of nuclear fission. The model treats the nucleus as an incompressible drop of nuclear fluid, similar to a drop of ordinary liquid. The basic idea is that nucleons (protons and neutrons) interact strongly with nearest neighbours, much like molecules in a liquid. Because the nuclear force is short-ranged and saturating, each nucleon interacts only with a limited number of surrounding nucleons. This behaviour leads to many nuclear properties resembling those of liquid droplets.

Physical Basis of the Model

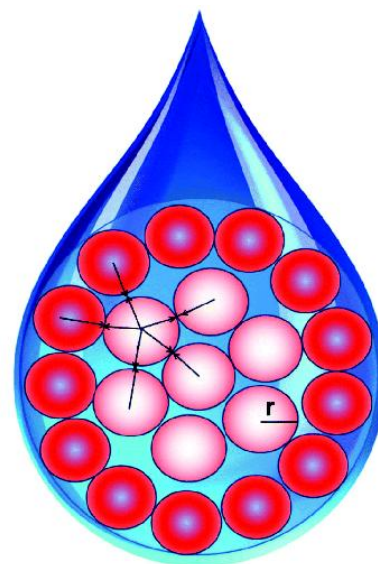


Figure 3. 2



1. Short-range nuclear force

Nuclear force acts only over distances of about 1–2 femtometers.

This local interaction is similar to the cohesive force between molecules in a drop.

2. Saturation of nuclear forces

Each nucleon interacts with only a few neighbours.

This leads to the approximate constancy of nuclear density and binding energy per nucleon.

3. Incompressibility

Nuclear matter has nearly constant density, independent of mass number.

This is similar to incompressible liquids.

4. Surface effects

Nucleons on the surface of the nucleus experience fewer attractive interactions.

This leads to a surface energy term, just like surface tension in liquids.

5. Competition between nuclear attraction and Coulomb repulsion

Protons repel each other due to Coulomb forces.

This competes with the attractive nuclear force and affects nuclear stability.

Semi-Empirical Mass Formula (Weizsäcker Formula)

The liquid drop model is mathematically expressed using the semi-empirical mass formula (SEMF). It calculates the binding energy of a nucleus using five main terms: Binding energy

$$B(A, Z) = \text{Volume term} - \text{Surface term} - \text{Coulomb term} - \text{Asymmetry term} + \text{Pairing term}$$

Explanation of Each Term

1. Volume term

Proportional to A (mass number).

Represents binding due to strong nuclear force acting on all nucleons.

Nucleons in the interior experience full binding, so this term increases total binding energy.

2. Surface term

Proportional to $A^{2/3}$.

Nucleons on the surface experience fewer interactions than those in the interior.

This term reduces binding energy, similar to surface tension in liquids.



3. Coulomb term

Proportional to $Z(Z-1)/A^{1/3}$.

Represents electrostatic repulsion between protons.

Reduces binding energy and explains why heavy nuclei become unstable.

4. Asymmetry term

Proportional to $(N - Z)^2 / A$.

Nuclei prefer $N \approx Z$ for stability.

This comes from the Pauli exclusion principle: too many neutrons or too many protons increases energy and reduces binding.

5. Pairing term

Depends on whether Z and N are even or odd.

Even–even nuclei have extra binding.

Odd–odd nuclei have reduced binding.

Even–odd and odd–even are intermediate.

This explains high stability of even–even nuclei.

Successes of the Liquid Drop Model

1. Explains average binding energy for medium and heavy nuclei.
2. Describes fission accurately.
3. Shows why very heavy nuclei (like uranium) can split easily.
4. Explains why nuclear density is constant.
5. Predicts general trends in nuclear stability.

Limitations

1. Cannot explain magic numbers.
2. Cannot describe fine nuclear structure.
3. Fails for light nuclei.
4. Does not explain spin and parity of nuclei.
5. Cannot predict detailed energy levels.

Because of these limitations, the liquid drop model is complemented by the shell model and collective model.

Nuclear Fission Explanation using Liquid Drop Model



The model gives an intuitive explanation of fission. A heavy nucleus is like a large charged liquid drop. If it is distorted (for example, by neutron absorption), the following occurs:

- Surface tension tries to restore spherical shape.
- Coulomb repulsion tries to push the nucleus apart.

In very heavy nuclei, the Coulomb repulsion becomes strong enough that a slight deformation causes the nucleus to split into two smaller nuclei. This is similar to a charged drop of liquid breaking into smaller drops. The liquid drop model predicts the energy release in fission and explains why nuclei like uranium-235 and plutonium-239 undergo fission more easily. The liquid drop model provides a macroscopic understanding of nuclear behaviour. It successfully explains bulk properties such as binding energy, nuclear stability, and fission. Although it cannot describe the microscopic details of individual nucleons, it forms the foundation for many nuclear theories and remains essential in nuclear physics.

Similarities between Nucleus and liquid drop

The liquid drop model is based on several important similarities between an atomic nucleus and an ordinary drop of liquid. These similarities justify treating the nucleus as a drop of incompressible nuclear fluid.

1. Short-range attractive forces

In a liquid drop, molecules attract each other only when they are very close.

Similarly, nuclear force between nucleons acts only at short range (about 1–2 fm).

This makes both systems cohesive only over small distances.

2. Saturation of forces

In a liquid, each molecule interacts strongly only with a small number of neighbours.

In a nucleus, each nucleon interacts with only a few nearest nucleons due to saturation of nuclear force.

This is why nuclear density remains nearly constant across all nuclei.

3. Constant density

A liquid drop has almost uniform density throughout its volume.

The nucleus also has nearly constant density, independent of its size.

This supports the idea of an incompressible nuclear fluid.



4. Surface effects

Molecules at the surface of a liquid experience fewer interactions, creating surface tension.

Nucleons at the nuclear surface also have fewer neighbours, which lowers their binding energy.

This leads to a surface energy term in the semi-empirical mass formula.

5. Balance between attractive and repulsive forces

In a liquid drop, cohesive forces hold the drop together, but repulsive forces (if the drop is charged) tend to disrupt it.

In a nucleus, attractive nuclear forces hold nucleons together, while electrostatic repulsion between protons tends to break the nucleus apart.

The competition between these forces determines stability in both systems.

6. Shape and deformation behaviour

A liquid drop tends to be spherical due to surface tension but can become deformed when disturbed.

A nucleus also tends to be spherical but can deform under excitation or external influence.

This deformation is important in explaining nuclear fission.

7. Fission-like behaviour

A highly charged liquid drop can break into smaller droplets.

Similarly, a heavy nucleus (such as uranium) can split into two smaller nuclei, demonstrating nuclear fission.

The model explains fission as a competition between surface tension and Coulomb repulsion.

The nucleus behaves like a liquid drop because both systems:

- have short-range saturated forces,
- maintain constant density,
- show surface effects,
- exhibit competition between attraction and repulsion,
- and can undergo deformation and division.

These similarities form the foundation of the liquid drop model in nuclear physics.

3.4 Mass formula Shell Model

The shell model was originally introduced to explain why certain nuclei show exceptional stability (the magic numbers). However, besides predicting spin, parity, and magnetic moments, the shell model also provides a **mass formula** that corrects the limitations of the liquid drop model. The semi-empirical mass

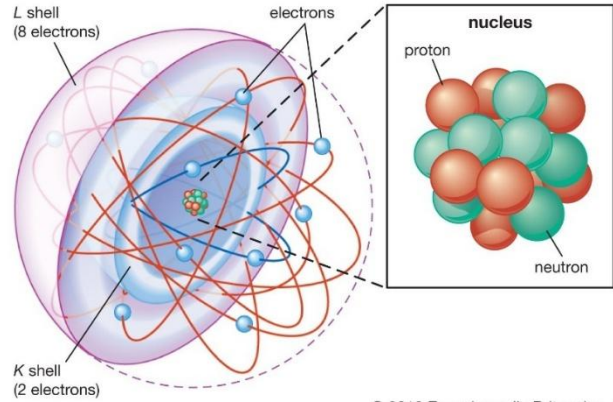


Figure 3. 3 (Shell Model of Nucleus)

formula (Weizsäcker formula) explains the bulk properties of nuclei but fails at magic numbers. Nuclei with closed shells show much higher stability and larger binding energy than predicted. To account for this, the shell model introduces a **shell correction** to the liquid drop mass formula. The idea is: Total binding energy = Liquid drop binding energy + Shell correction. This leads to the concept of the shell model mass formula. Treat the nucleus as A nucleons packed together. The total **binding energy** $B(A, Z)$ (energy needed to separate all nucleons) is modeled as a sum of contributions:

$$B(A, Z) = B_{\text{vol}} + B_{\text{surf}} + B_{\text{Coul}} + B_{\text{asym}} + B_{\text{pair}}$$

Each term has a clear physical origin.

Each nucleon interacts with a roughly constant number of nearest neighbours \rightarrow binding energy proportional to number of nucleons A .

$$B_{\text{vol}} = a_v A$$

a_v is positive (≈ 15 – 16 MeV empirically).

Nucleons at the surface have fewer neighbors, so reduce binding \propto surface area $\sim A^{2/3}$.

$$B_{\text{surf}} = -a_s A^{2/3}$$

a_s (≈ 17 – 19 MeV) subtracts from volume binding.

Protons repel each other electrostatically; for a uniformly charged sphere of radius R the self-energy is $\frac{3}{5} \frac{e^2 Z^2}{R}$. Use $R = r_0 A^{1/3}$.



$$B_{\text{Coul}} \approx -\frac{3}{5} \frac{e^2 Z^2}{r_0 A^{1/3}}$$

Usually written as

$$B_{\text{Coul}} = -a_c \frac{Z(Z-1)}{A^{1/3}}$$

- factor $Z(Z-1)$ corrects for self-interaction (or sometimes Z^2 is used for simplicity).
- $a_c \approx 0.6 - 0.8 \text{ MeV}$

Protons and neutrons fill quantum states; if $N \neq Z$, one species fills higher momentum states \Rightarrow extra kinetic energy. Model each as a Fermi gas in the same volume.

Sketch of derivation (clean and important):

Let N neutrons and Z protons with $A = N + Z$. For a noninteracting Fermi gas in volume V , kinetic energy per species scales like $N^{5/3}/V^{2/3}$. Since $V \propto A$ (roughly), the total kinetic energy part scales like

$$E_k \propto \frac{N^{5/3} + Z^{5/3}}{A^{2/3}}.$$

Write $N = \frac{A}{2} + x$, $Z = \frac{A}{2} - x$ where $x = \frac{N-Z}{2}$. Expand using binomial series:

$$\left(\frac{A}{2} \pm x\right)^{5/3} \approx \left(\frac{A}{2}\right)^{5/3} \pm \frac{5}{3} \left(\frac{A}{2}\right)^{2/3} x + \frac{5}{9} \left(\frac{A}{2}\right)^{-1/3} x^2 + \dots$$

Adding the two gives cancellation of odd terms, leaving a leading x^2 correction:

$$N^{5/3} + Z^{5/3} \approx 2 \left(\frac{A}{2}\right)^{5/3} + \frac{10}{9} \left(\frac{A}{2}\right)^{-1/3} x^2 + \dots$$

Thus the **extra** kinetic energy due to neutron-proton imbalance scales as $x^2/A^{1/3}$ divided by $A^{2/3}$ from the volume factor \rightarrow overall $\propto x^2/A$. Substituting $x = (N-Z)/2$ gives $(N-Z)^2/A$, or in terms of A and Z , $(A-2Z)^2/A$.

So we write:

$$B_{\text{asym}} = -a_a \frac{(N-Z)^2}{A} = -a_a \frac{(A-2Z)^2}{A}$$



with a_a (asymmetry coefficient) $\approx 23\text{--}25$ MeV.

nucleon pairing (like BCS) gives extra binding for even-even nuclei and less binding for odd-odd. Empirical form:

$$B_{\text{pair}} = \begin{cases} +\frac{a_p}{A^{1/2}} & \text{even } Z, \text{ even } N \\ 0 & \text{odd } A \\ -\frac{a_p}{A^{1/2}} & \text{odd } Z, \text{ odd } N \end{cases}$$

Typical $a_p \approx 11\text{--}12$ MeV. (Sign: + for even–even, – for odd–odd.)

SEMF final form

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

where δ is the pairing term above.

And the nuclear mass (rest energy) relates to binding energy:

$$M(A, Z)c^2 = Zm_p c^2 + Nm_n c^2 - B(A, Z)$$

(For **atomic** masses, include electron masses and electron binding energy corrections.)

Typical empirical coefficients

A commonly used fit gives approximately:

- $a_v \approx 15.5$ MeV
- $a_s \approx 17.5$ MeV
- $a_c \approx 0.7$ MeV
- $a_a \approx 23$ MeV
- $a_p \approx 11$ MeV

(These come from fits to measured nuclear masses.)

- SEMF captures bulk trends (why binding per nucleon peaks near Fe).
- Misses shell effects (magic numbers), deformation, Wigner term, microscopic corrections.
- Shell corrections (Strutinsky), mean-field and pairing models, or full microscopic models (Hartree–Fock, DFT) improve agreement.



Need for Shell Model Mass Formula

1. The liquid drop model predicts smooth variation of binding energy per nucleon.
2. Experimental binding energy shows sudden jumps at magic numbers (2, 8, 20, 28, 50, 82, 126).
3. These jumps cannot be explained unless nucleons occupy discrete energy levels (shells).
4. Therefore, the mass formula must include terms that reflect shell structure.

Liquid drop model energy + Energy due to filling of shells (shell correction)

Mathematically written as:

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c Z^2 / A^{1/3} - a_a (N - Z)^2 / A + \delta + E_{\text{shell}}$$

Where

E_{shell} is the additional energy that accounts for shell effects.

Meaning of the Shell Correction Term

E_{shell} is not one single term. It represents the difference between the energy predicted by the liquid drop model and the actual experimental energy due to closed shells.

General behaviour of E_{shell} :

- For magic number nuclei, E_{shell} is strongly negative (which means extra binding, nucleus is more stable).
- For nuclei away from magic numbers, E_{shell} is small or zero. (shell effects fade away).

Thus, shell effects lower the energy of closed-shell configurations, producing peaks in binding energy.

They arise because nucleons are not uniformly distributed.

Instead, they occupy discrete energy levels in a central potential (typically a harmonic oscillator with spin-orbit coupling).

When a shell closes:

- the configuration becomes extremely stable,
- the total energy of the nucleus drops noticeably,
- experimental mass and binding energy shift away from liquid drop predictions.

Therefore, the shell correction fixes this deviation.

Major Contributions to Shell Model Mass Formula



1. Mean-field potential energy
Nucleons move independently in an average potential well.
2. Spin-orbit coupling
Strong spin-orbit interaction splits energy levels, creating observed magic numbers.
3. Pairing energy
Even-even nuclei gain extra binding due to paired nucleons.
This is consistent with the pairing term in the mass formula.
4. Shell closure energy
Distinct extra stability for closed shells.

Significance of the Shell Model Mass Formula

1. Explains why magic number nuclei have higher binding energy.
2. Corrects the smooth curve of the liquid drop model.
3. Predicts nuclear masses more accurately.
4. Explains why certain isotopes are more stable even when the liquid drop model predicts instability.
5. Essential for modelling nuclear fission and fusion.
6. Helps in understanding super heavy elements and island of stability.

Simple Interpretation

Liquid drop model gives the smooth background binding energy.

The shell model adds fine corrections (like quantum energy levels).

Together, they provide the most accurate description of nuclear masses.

3.5 Magic numbers

In nuclear physics, **magic numbers** are specific numbers of **protons (Z)** or **neutrons (N)** that make the nucleus *extra stable*. This happens because nuclear energy levels (shells) get completely filled — like noble gas configuration in atoms, but for nuclei. These numbers pop out naturally from the **Nuclear Shell Model**.

Standard Magic Numbers

The experimentally observed magic numbers are:

2, 8, 20, 28, 50, 82, 126

- For **protons**: $Z = 2, 8, 20, 28, 50, 82$



- For **neutrons**: $N = 2, 8, 20, 28, 50, 82, 126$

Examples of nuclei having magic N or Z :

- **He-4**: $Z=2, N=2$ (double magic)
- **O-16**: $Z=8, N=8$ (double magic)
- **Ca-40**: $Z=20, N=20$ (double magic)
- **Pb-208**: $Z=82, N=126$ (double magic, ultra-stable)

The simple harmonic oscillator or square-well model alone *cannot* explain magic numbers like 2, 8, 20, 28, 50, 82, 126.

The correct explanation comes when you include:

1) Spin–Orbit Coupling

A strong $\mathbf{l} \cdot \mathbf{s}$ interaction splits levels into:

$$j = l + \frac{1}{2} \text{ and } j = l - \frac{1}{2}$$

This splitting rearranges level ordering so that certain shells close exactly at the magic numbers.

2) Energy Gaps

Magic numbers correspond to large energy gaps between filled and next unfilled shells — giving extra binding.

Experimental Evidence for Magic Numbers

Magic numbers show up in multiple ways:

1) Higher binding energy per nucleon

Nuclei with magic numbers have unusually large binding energies.

2) Higher first excitation energy

More energy is required to excite the nucleus (closed shell \rightarrow rigid).

3) Low neutron capture cross section

Magic nuclei don't like absorbing extra neutrons.

4) Greater natural abundance

Example: ^{40}Ca ^{208}Pb are very stable and abundant.

5) Systematic irregularities

Sudden jumps in:

- Separation energy



- Quadrupole moments
- Nuclear radii

Double Magic Nuclei

When both Z and N are magic \rightarrow *super* stable.

Examples:

- **He-4**
- **O-16**
- **Ca-40, Ca-48**
- **Ni-56**
- **Sn-100**
- **Pb-208**

These nuclei often have spherical shapes because closed shells resist deformation.

Some isotopes far from stability show **new magic numbers** (like $N = 14, 16, 32$).

These come from:

- changes in shell ordering
- weak binding in neutron-rich nuclei

But the classical magic numbers remain the foundation.

3.6 Evidences that led to shell

The development of the nuclear shell model was driven by several experimental observations that strongly suggested nuclei are not just uniform drops of nuclear matter, but systems with quantized energy levels similar to atomic shells. Researchers noticed unusual patterns in stability, binding energy, separation energy, nuclear spins, and excitation spectra that could not be explained by the liquid-drop model alone. The striking behaviour of nuclei with certain neutron or proton numbers—later called *magic numbers*—along with abrupt changes in nuclear properties, provided compelling evidence that nucleons occupy discrete shells. These anomalies collectively pushed physicists to propose the shell structure of the nucleus, laying the foundation for the modern shell model.

1) Magic Numbers (the biggest giveaway)

Some nuclei with N or $Z = 2, 8, 20, 28, 50, 82, 126$ were suddenly **more stable** than neighbors.



Strange observations:

- higher binding energy
- more abundant naturally
- stronger resistance to deformation
- high first excited state energies

These sudden jumps screamed:

This was the *first and strongest hint*.

2) Abrupt Changes in Separation Energy

Plot neutron separation energy S_n vs. neutron number —

At magic N, there's a **big drop** in separation energy.

Meaning:

Once a shell is filled, adding one more neutron is suddenly harder.

This behaviour is **impossible** to explain using:

- liquid drop model
- collective model (without shells)

3) Low Neutron-Capture Cross Sections (Magic nuclei hate extra neutrons)

Magic nuclei (like Pb-208) show **very low** neutron capture probability.

Why?

Closed shells = stable AF = don't want extra nucleons.

This selective resistance again signaled *shell closures*

4) Higher Excitation Energies for Magic Nuclei

Magic-number nuclei have:

- very large first excited state energies
- sparse level spacing

Which means:

Closed shells → rigid structure → hard to excite.

This behaviour mimics noble gases, pushing the analogy further.

5) Nuclear Spin & Parity Patterns

Experimental measurements of nuclear spin/parity (from hyperfine structure, transfer reactions) showed:

- Odd-A nuclei often have spin values that match *single-particle* predictions
- Even-even nuclei → spin = 0^+ (super stable)



- Odd-odd nuclei → more complex spins

These matched perfectly with **one unpaired nucleon in a specific shell** → direct confirmation of shell structure.

6) Isotope Abundance Patterns

Certain isotopes (Ca-40, Ca-48, Sn-120, Pb-208) appear much more in nature because they're **double magic**.

This mass abundance wasn't random — it was **shell-driven**.

7) Quadrupole Moments (Shape Clues)

Magic nuclei have:

- very small electric quadrupole moments
→ meaning they're extremely spherical.

Shell closures → no deformation.

This observation directly supported “closed configurations.”

8) Systematic Fits to Nuclear Mass (Liquid Drop Model Failed)

The semi-empirical mass formula *missed* many nuclei by a wide margin.

Where?

Exactly at magic numbers.

So scientists added a “**shell correction term**” → only way the math made sense.

This historical failure exposed the need for shell structure.

9) Energy Level Spacings in Nuclear Spectra

When nuclei were bombarded with:

- neutrons
- protons
- alpha particles

The resulting spectra showed **grouped energy levels** — not continuous.

These groups looked suspiciously like **atomic shells** → **nuclear analogy confirmed**.



Unit 4: Radioactivity

1. Discovery of Radioactivity
2. Exponential decay law
3. Half life, Mean life
4. Natural and artificial radio activity
5. Properties of rays
6. Gamow's theory of alpha decay
7. Geiger Nuttal law
8. beta decay Spectra

4.1 Discovery of Radioactivity

Radioactivity is a natural nuclear phenomenon in which unstable atomic nuclei spontaneously emit radiation in order to attain a more stable configuration. This was first discovered by Henri Becquerel in 1896 while studying uranium salts. Later, Marie Curie and Pierre Curie conducted extensive research and identified several radioactive elements.

Certain atomic nuclei possess an unfavorable ratio of neutrons to protons or contain excess nuclear energy. Due to this, the attractive nuclear forces cannot balance the repulsive electrostatic forces between protons. As a result, these nuclei become unstable. To achieve stability, they undergo spontaneous disintegration, during which particles or electromagnetic radiation are emitted. This process is called radioactivity, and the emitted rays are known as radioactive radiations.

Radioactivity is strictly a nuclear phenomenon; it is not influenced by external factors such as temperature, pressure, or chemical reactions. It depends solely on the internal structure and energy state of the nucleus.

Discovery of Radioactivity

The discovery of radioactivity was not a sudden event but the result of a series of observations made by physicists studying the interaction between matter and energy. The major contribution came from Henri Becquerel, followed by extensive research by Marie Curie and Pierre Curie. The process can be understood in three stages:



1. Background Work Before the Discovery

During the late 19th century, scientists were fascinated by the discovery of X-rays by Wilhelm Conrad Roentgen (1895). Roentgen had shown that X-rays could pass through materials and expose photographic plates.

This discovery sparked interest among scientists, including Henri Becquerel, who began investigating whether materials that absorb sunlight could also produce X-ray-like radiation.

Becquerel believed that fluorescent and phosphorescent materials, after absorbing sunlight, might emit some unknown penetrating radiation similar to X-rays.

2. Henri Becquerel's Experiments (1896)

(a) Initial Hypothesis

Becquerel chose **uranium salts**, which are known to be phosphorescent, to test whether sunlight could make them emit X-ray-like rays.

He placed a photographic plate wrapped in black paper and kept uranium salt crystals on top of it. His expectation: the crystals must absorb sunlight and then emit radiation strong enough to penetrate the paper and darken the photographic plate.

(b) Cloudy Weather Incident

Becquerel planned to expose the uranium salts to sunlight. But due to cloudy weather in Paris for several days, he could not expose the samples to sunlight. He kept the covered photographic plate with uranium salts aside in a drawer, assuming no result would be obtained.

(c) Unexpected Observation

When he developed the photographic plate later, he found that it was darkened with clear impressions even though:

- the uranium salts were **not exposed to sunlight**,
- and the photographic plate was wrapped in thick black paper.

This showed that uranium salts were emitting some kind of penetrating radiation **spontaneously**, without any external stimulus.

(d) Conclusion

Becquerel concluded that the radiation emitted by uranium was:

- a property of the uranium atom itself, not due to fluorescence,



- spontaneously emitted,
- capable of penetrating opaque materials,
- and capable of ionizing photographic plates.

This was the first evidence of **natural radioactivity**.

3. Contribution of Marie Curie and Pierre Curie

After Becquerel's discovery, Marie Curie decided to study this new kind of radiation in detail.

(a) Term "Radioactivity"

Marie Curie coined the term **radioactivity** to describe the spontaneous emission of radiation from elements.

(b) Systematic Study

Marie Curie examined various minerals and observed that:

- uranium and thorium both showed spontaneous radiation,
- intensity of radiation depended on the amount of radioactive element present,
- radiation was a property of the atom, not the molecule.

This proved that radioactivity originates from the **atomic nucleus**, not from chemical combinations.

(c) Discovery of New Radioactive Elements

Marie and Pierre Curie discovered two new highly radioactive elements:

1. **Polonium** (named after Curie's home country, Poland)
2. **Radium**

Radium, in particular, showed extremely strong radiation, far more intense than uranium.

Their discoveries confirmed that radioactivity was not an isolated property of uranium but a general nuclear property present in certain heavy elements.

4. Key Experimental Findings

1. Radioactive rays could ionize air.
2. They could expose photographic plates.
3. They were unaffected by temperature, pressure, or chemical changes.
4. Radioactivity was directly proportional to the quantity of the radioactive element.
5. The radiation came from the **nucleus of the atom**.

5. Nobel Prize Recognition



- **1903 Nobel Prize in Physics:**

Awarded jointly to Henri Becquerel, Marie Curie, and Pierre Curie for their pioneering work on radioactivity.

- **1911 Nobel Prize in Chemistry:**

Awarded solely to Marie Curie for the discovery of polonium and radium.

These achievements solidified radioactivity as a major area of nuclear physics and chemistry.

Nature and Types of Radioactive Emissions

Radioactive decay commonly occurs in three forms:

1. Alpha Decay (α -decay)

- An alpha particle consists of two protons and two neutrons (same as a helium nucleus).
- An unstable heavy nucleus ejects an alpha particle.
- After emission, the daughter nucleus has:
 - Atomic number decreased by 2
 - Mass number decreased by 4

Alpha particles have low penetrating power but high ionizing power. Common in heavy nuclei like uranium, thorium, and radium.

2. Beta Decay (β -decay)

Beta particles are high-speed electrons or positrons emitted from the nucleus.

There are two types:

- β^- decay: A neutron transforms into a proton + electron + antineutrino.
 - Atomic number increases by 1; mass number remains unchanged.
- β^+ decay: A proton transforms into a neutron + positron + neutrino.
 - Atomic number decreases by 1; mass number remains unchanged.

Beta particles have moderate penetrating power and moderate ionizing power.

3. Gamma Decay (γ -decay)

- Gamma rays are high-energy electromagnetic waves emitted when the nucleus drops from an excited state to a lower energy state.
- No change in atomic number or mass number.
- Very high penetrating power and low ionizing power.

Gamma radiation generally accompanies alpha or beta emission.



Cause of Radioactivity

Radioactivity arises because some nuclei are inherently unstable. The primary reasons include:

1. **Incorrect neutron–proton ratio.**
2. **Excess internal nuclear energy.**
3. **Too many nucleons (massive nuclei) causing imbalance between nuclear force and electrostatic repulsion.**

When a nucleus becomes unstable, it tends to undergo decay until a stable daughter nucleus is formed.

Characteristics of Radioactivity

1. Radioactivity is a spontaneous process and cannot be controlled externally.
2. It is a nuclear phenomenon and independent of temperature, pressure, and chemical state.
3. Each radioactive element has a definite rate of decay.
4. The decay of a large number of nuclei follows statistical laws.
5. Radioactivity decreases exponentially with time.
6. Radioactive decay continues until a stable isotope forms.
7. Activity of a radioactive sample is measured in becquerel (Bq) or curie (Ci).

Applications of Radioactivity

1. Medical applications: cancer treatment (radiotherapy), medical imaging.
2. Carbon dating: estimating age of fossils.
3. Nuclear power generation.
4. Industrial applications: thickness measurement, leak detection.
5. Food irradiation: sterilizing food and medical instruments.
6. Tracing chemical and biological processes using radioisotopes.

Exponential decay law

Radioactive decay is a spontaneous nuclear process in which unstable nuclei disintegrate into more stable nuclei by emitting radiation. Although it is impossible to predict which individual nucleus will decay at a specific moment, the **overall decay rate of a large number of nuclei follows a definite mathematical law**. This is known as the **exponential**



decay law. This law forms the foundation of all radioactive measurements such as half-life, mean life, and activity.

1. Basic Idea Behind the Law

Every radioactive nucleus has a certain probability of decaying per unit time. This probability is constant and does not change with:

- time,
- temperature,
- pressure,
- or any physical or chemical condition.

Let N be the number of radioactive nuclei present at time t .

The decrease in N over a very small time interval dt is proportional to the number of nuclei present at that moment.

Thus, the more nuclei present, the faster the sample will decay.

2. Mathematical Form: Differential Equation

The law can be expressed as:

$$dN/dt \propto N$$

but since N decreases with time:

$$dN/dt = -\lambda N$$

Where:

N = number of undecayed nuclei at time t

λ = decay constant (probability of decay per nucleus per second)

The negative sign indicates a decrease in N with time.

3. Solving the Differential Equation (Derivation of Exponential Law)

We solve the equation:

$$dN/dt = -\lambda N$$

Rewriting:

$$dN/N = -\lambda dt$$

Integrating both sides:

$$\int (1/N) dN = -\lambda \int dt$$

$$\ln N = -\lambda t + C$$

Where C is the constant of integration.



To find C, use the initial condition:

At $t = 0$, $N = N_0$ (initial number of nuclei)

So,

$$\ln N_0 = C$$

Substitute back:

$$\ln N = -\lambda t + \ln N_0$$

Rewriting:

$$\ln (N / N_0) = -\lambda t$$

Taking exponential on both sides:

$$N = N_0 e^{-\lambda t}$$

This is the **Exponential Decay Law**.

4. Meaning of the Formula

The equation $N = N_0 e^{-\lambda t}$ states:

- The number of radioactive nuclei decreases exponentially with time.
- The decay is not linear; instead, it becomes slower as N decreases.
- Even though nuclei keep decaying, N never becomes zero mathematically.

It continuously approaches zero as $t \rightarrow \infty$.

5. Activity (Rate of Decay)

Activity A is the number of decays per second:

$$A = -dN/dt$$

From the basic decay law:

$$A = \lambda N$$

Substituting $N = N_0 e^{-\lambda t}$

$$A = A_0 e^{-\lambda t}$$

$$\text{where } A_0 = \lambda N_0$$

Thus, activity also decreases exponentially.

6. Decay Constant (λ)

λ is the probability per unit time that a single nucleus will decay.

Units of λ :



- s^{-1} (if time in seconds)
- min^{-1} (if time in minutes)
- $year^{-1}$ (if time in years)

Higher $\lambda \rightarrow$ faster decay

Lower $\lambda \rightarrow$ slower decay

7. Graphical Representation of Exponential Decay

If N is plotted against t , the curve is:

- continuously decreasing,
- steeper at the beginning,
- gradually flattening as time increases,
- never touching the time axis (as N never becomes zero).

This is called an exponential decay curve.

8. Important Consequences of Decay Law

1. Radioactivity is entirely statistical in nature.
2. Large samples follow the law very accurately.
3. Decay rate depends only on λ , not on external conditions.
4. The law forms the basis of half-life and mean life calculations.

9. Relationship with Half-Life and Mean Life

Using the decay law, we obtain:

$$\text{Half-life } (t_{1/2}) = 0.693 / \lambda$$

$$\text{Mean life } (\tau) = 1 / \lambda$$

Also,

$$\tau = 1.44 \times t_{1/2}$$

These formulas originate directly from $N = N_0 e^{-\lambda t}$

4.3 Half life, Mean life



Let $N(t)$ be the number of undecayed nuclei at time t . The exponential decay law is

$$N(t) = N_0 e^{-\lambda t},$$

where N_0 is the number at $t = 0$ and λ is the decay constant (probability of decay per nucleus per unit time).

Also activity $A(t)$, the rate of decay, is

$$A(t) = -\frac{dN}{dt} = \lambda N(t) = \lambda N_0 e^{-\lambda t}.$$

1. Half-life ($t_{1/2}$) — derivation

Definition: half-life $t_{1/2}$ is the time required for the number of undecayed nuclei to reduce to half its initial value:

$$N(t_{1/2}) = \frac{N_0}{2}.$$

Start from the decay law:

$$N(t_{1/2}) = N_0 e^{-\lambda t_{1/2}} = \frac{N_0}{2}.$$

Divide both sides by N_0 :

$$e^{-\lambda t_{1/2}} = \frac{1}{2}.$$

Take natural logarithm (\ln) on both sides:

$$-\lambda t_{1/2} = \ln \left(\frac{1}{2} \right).$$

Use $\ln(1/2) = -\ln 2$:

$$-\lambda t_{1/2} = -\ln 2 \Rightarrow \lambda t_{1/2} = \ln 2.$$

Solve for $t_{1/2}$:

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

Since $\ln 2 \approx 0.693$, an often used form is



$$t_{1/2} = \frac{0.693}{\lambda}.$$

Comment: $t_{1/2}$ is independent of N_0 and external conditions.

2. Mean life (average lifetime) τ — derivation

Definition: mean life τ is the average time a nucleus survives before decaying. It is the expectation value of the lifetime of a nucleus.

Treat the decay as a probability distribution for lifetime t . The probability density that a nucleus decays between t and $t + dt$ is

$$p(t) dt = \frac{A(t) dt}{N_0} = \frac{\lambda N_0 e^{-\lambda t} dt}{N_0} = \lambda e^{-\lambda t} dt.$$

Thus $p(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, which is an exponential probability density with parameter λ .

The mean life is the expectation value:

$$\tau = \langle t \rangle = \int_0^{\infty} t p(t) dt = \int_0^{\infty} t \lambda e^{-\lambda t} dt.$$

Evaluate the integral. Compute $\int_0^{\infty} t e^{-\lambda t} dt$. Use integration by parts or a standard integral:

Let $= \int_0^{\infty} t e^{-\lambda t} dt$.

Integration by parts with $u = t$, $dv = e^{-\lambda t} dt$ gives:

$$u = t \Rightarrow du = dt, dv = e^{-\lambda t} dt \Rightarrow v = -\frac{1}{\lambda} e^{-\lambda t}.$$

So

$$I = uv \Big|_0^{\infty} - \int_0^{\infty} v du = \left[-\frac{t}{\lambda} e^{-\lambda t} \right]_0^{\infty} + \frac{1}{\lambda} \int_0^{\infty} e^{-\lambda t} dt.$$

The boundary term $\left[-\frac{t}{\lambda} e^{-\lambda t} \right]_0^{\infty} = 0$ because $te^{-\lambda t} \rightarrow 0$ as $t \rightarrow \infty$, and at $t = 0$ it is zero. The remaining integral is

$$\frac{1}{\lambda} \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda} \cdot \frac{1}{\lambda} = \frac{1}{\lambda^2}.$$



Thus $I = \frac{1}{\lambda^2}$. Returning to τ :

$$\tau = \lambda \cdot I = \lambda \cdot \frac{1}{\lambda^2} = \frac{1}{\lambda}.$$

Therefore

$$\boxed{\tau = \frac{1}{\lambda}}$$

3. Relation between mean life and half-life

We have

$$t_{1/2} = \frac{\ln 2}{\lambda}, \tau = \frac{1}{\lambda}.$$

Eliminate λ :

$$\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2} \Rightarrow \tau = \frac{t_{1/2}}{\ln 2}.$$

Numerically $\frac{1}{\ln 2} \approx 1.4427$, so

$$\boxed{\tau \approx 1.44 t_{1/2}}.$$

4. Alternative expression using activity

Since $A(t) = \lambda N(t)$, the initial activity is $A_0 = \lambda N_0$. One can express half-life and mean life in terms of A_0 and λ as:

$$t_{1/2} = \frac{\ln 2}{\lambda}, \tau = \frac{1}{\lambda}.$$

These are independent of sample size.

If $\lambda = 0.01 \text{ s}^{-1}$:

- Mean life $\tau = 1/\lambda = 100 \text{ s}$.
- Half-life $t_{1/2} = \ln 2/\lambda = 0.693/0.01 = 69.3 \text{ s}$.

Note: $\tau \approx 1.44 \times 69.3 \text{ s} \approx 100 \text{ s}$.

4.3 Natural and artificial radio activity



Radioactivity is the spontaneous emission of alpha, beta or gamma radiation from unstable nuclei. Based on its origin, radioactivity is classified into two types: **natural radioactivity** and **artificial radioactivity**.

1. Natural Radioactivity

Natural radioactivity is the phenomenon in which certain elements present in nature emit radiation **spontaneously** without any external cause. The nuclei are inherently unstable due to an improper neutron-proton ratio.

Discovery

Henri Becquerel (1896) discovered natural radioactivity while studying uranium salts. Marie Curie later confirmed that thorium and new elements like polonium and radium also show natural radioactivity.

Examples of naturally radioactive elements

Uranium (U)
Thorium (Th)
Radium (Ra)
Actinium (Ac)
Polonium (Po)
Carbon-14 (weak radioactive isotope)

Characteristics

1. Radiation is emitted spontaneously and cannot be stopped or controlled.
2. The rate of decay is independent of temperature, pressure, or chemical changes.
3. Elements generally belong to heavy nuclei with atomic number above 82.
4. These elements form natural radioactive decay series such as Uranium series, Thorium series, Actinium series.
5. The decay continues until a stable, non-radioactive daughter product (like lead isotopes) is formed.

Types of radiations emitted naturally

Alpha rays (helium nuclei)
Beta rays (electrons or positrons)
Gamma rays (high-energy photons)



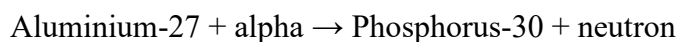
2. Artificial Radioactivity (Induced Radioactivity)

Artificial radioactivity is the phenomenon in which a stable nucleus becomes radioactive after being bombarded by external particles such as neutrons, protons, alpha particles or gamma rays. The radioactive behavior is induced artificially in a laboratory or nuclear reactor.

Discovery

Artificial radioactivity was discovered by Irène Curie and Frédéric Joliot (1934). They bombarded aluminium with alpha particles and produced radioactive phosphorus. This was the first artificial radioactive isotope ever created.

Example reaction:



P-30 is radioactive and undergoes beta decay.

Process of artificial radioactivity

1. A stable nucleus is targeted with external particles.
2. The nucleus absorbs the particle and forms an excited, unstable isotope.
3. This new nucleus decays by emitting alpha, beta or gamma radiation.

Common particles used in bombardment

Neutrons (most common due to zero charge)

Alpha particles

Protons

Deuterons

Gamma photons

Examples of artificially produced radioactive isotopes

Phosphorus-32

Iodine-131

Cobalt-60

Carbon-11

Technetium-99m

These isotopes are used in medicine, industry and research.



Characteristics

1. Radioactivity is produced only after bombardment by external particles.
2. It can be controlled by controlling the particle beam.
3. Produces isotopes not found naturally.
4. Used in nuclear medicine, cancer therapy, radiography, thickness measurements, food irradiation, and scientific research.

Natural Radioactivity	Artificial Radioactivity
Occurs spontaneously in nature	Produced by bombarding stable nuclei
Present in heavy elements like U, Th, Ra	Can be induced in many stable elements
Cannot be controlled	Can be controlled by adjusting the beam
Exists without human intervention	Requires nuclear reactors or accelerators
Limited number of naturally radioactive elements	Unlimited artificially produced isotopes possible

Applications of Artificial Radioactivity

1. Medical diagnosis and therapy (I-131, Tc-99m).
2. Cancer treatment using Cobalt-60 gamma rays.
3. Industrial radiography for testing metal components.
4. Agriculture: mutation breeding, tracer studies.
5. Carbon dating and tracing metabolic pathways.

4.3 Properties of rays

Alpha rays (α -rays)

Origin and physical nature

- Alpha particles are identical to helium nuclei: two protons and two neutrons (${}^2\text{He}^4$), net charge $+2e$ and rest mass ≈ 4 u.
- Emitted by heavy, proton-rich unstable nuclei (for example, U, Th, Ra, Po) as a mode of decay that reduces both mass and atomic number:
Parent \rightarrow Daughter + α (mass number decreases by 4, atomic number decreases by 2).



- Emission is a nuclear process governed by quantum tunnelling through the nuclear potential barrier.

Energy and spectrum

- Alpha emissions are essentially monoenergetic for a given parent \rightarrow daughter transition (discrete energies), typically in the range of about 4–9 MeV for common α -emitters.
- Because the energy levels of nuclei are discrete, α -particle energies appear as sharp lines in an energy spectrum.

Interaction with matter (ionisation & stopping)

- Very large mass and +2 charge \rightarrow very high rate of energy loss per unit path length (high linear energy transfer, high LET).
- Ionisation: produces dense ion pairs along a short track; thus very strong ioniser.
- Penetration: very poor. Typical range in air is a few centimetres (depends on energy); stopped by a few micrometres of solid material (paper, outer dead layer of skin) or thin metal foil.
- Energy loss mechanism: mainly Coulomb interactions with atomic electrons (excitation and ionisation). At the end of range α -particles deposit remaining energy in a Bragg peak.

Deflection by fields

- Strongly deflected by electric and magnetic fields because of their charge; deflection direction is toward the negative plate.

Detection

- Easily detected by scintillation detectors (ZnS), surface barrier semiconductor detectors, ionisation chambers, and photographic plates (historical). Cloud chambers and bubble chambers show distinct short dense tracks.

Biological and safety aspects

- Externally low hazard (stopped by skin), but high hazard if α -emitters are ingested or inhaled because of high localised energy deposition \rightarrow severe tissue damage and high effective dose.



Beta rays (β -rays)

Origin and physical nature

- Beta radiation consists of high-speed electrons (β^-) or positrons (β^+) emitted from the nucleus during transformation of a nucleon:
 β^- : neutron \rightarrow proton + e^- + $\bar{\nu}$ (antineutrino)
 β^+ : proton \rightarrow neutron + e^+ + ν (neutrino)
- Also related process: electron capture, where an orbital electron is captured by the nucleus converting a proton to a neutron and emitting a neutrino; this often produces X-rays or Auger electrons subsequently.

Energy and spectrum

- Continuous energy spectrum from 0 up to a maximum end-point energy (Q-value) for a given transition. The continuous nature is explained by the sharing of decay energy between the beta particle and the (anti)neutrino.
- Typical β energies are in the keV to MeV range (commonly up to a few MeV).

Interaction with matter (ionisation & stopping)

- Small mass and single charge \rightarrow lower LET than α but higher than γ (moderate ionisation).
- Penetration: greater than α ; typical penetration in matter is millimetres to centimetres depending on energy. A few millimetres of aluminium or plastic can stop many β particles.
- Energy loss: via Coulomb interactions with electrons (ionisation and excitation) and radiative losses (bremsstrahlung), the latter becoming significant at higher β energies and in high-Z materials.
- Range is not strictly fixed because of spectrum; practical range depends on initial energy.

Deflection by fields

- Strongly deflected by electric and magnetic fields; β^- are deflected toward the positive plate, β^+ toward the negative plate. Direction and curvature used historically to identify charge.

Specific phenomena



- Positron emission is followed by annihilation of e^+ with an e^- producing two 511 keV γ -photons emitted approximately 180° apart — exploited in positron emission tomography (PET).
- Continuous spectrum and presence of neutrino were key observations leading to the neutrino hypothesis.

Detection

- Geiger–Müller counters, scintillation detectors, semiconductor detectors, cloud/bubble chambers (thin tracks), and proportional counters.

Biological and safety aspects

- Can penetrate skin and deposit energy in tissues—internal and external exposure are important. Bremsstrahlung generated when β interacts with high-Z shielding must be considered (use low-Z materials like plastic or aluminium as β shield, followed by a high-Z layer if needed to absorb bremsstrahlung).

Gamma Rays

Origin and physical nature

- Gamma rays are high-energy photons (electromagnetic radiation) emitted by excited nuclei as they transition to lower energy states. They carry no charge and have no rest mass.
- Gamma emission commonly accompanies α or β decay when the daughter nucleus is left in an excited state. Internal conversion is an alternative in which energy is transferred to an orbital electron which is ejected instead of a γ -photon.

Energy and spectrum

- Energies are discrete for nuclear transitions (keV to several MeV); spectral lines are characteristic of nuclear energy level differences.
- Because γ are photons, their energy is often denoted in electronvolts (keV, MeV).

Interaction with matter (attenuation & processes)

Three primary interaction mechanisms (dominant depending on photon energy and target Z):

1. Photoelectric absorption: γ photon completely absorbed, ejecting a bound electron. Dominant at low γ energies and in high-Z materials.



2. Compton scattering: inelastic scattering from quasi-free electrons, transferring part of the photon energy to the electron; dominant at intermediate energies.
3. Pair production: at energies >1.022 MeV, photon converts to e^+e^- pair in the Coulomb field of a nucleus; probability increases with photon energy and Z .
- Attenuation law (exponential): the intensity I of a monoenergetic γ beam after thickness x of material is

$$I = I_0 e^{(-\mu x)}$$

where μ is the linear attenuation coefficient (depends on material and energy).

- Half-value layer (HVL): thickness that reduces intensity to one half; $HVL = \ln 2 / \mu$.

Penetration and shielding

- High penetrating power; requires dense, high- Z materials (lead, tungsten) or several tens of centimetres of concrete for effective shielding depending on energy.
- Shielding design must consider secondary radiation (e.g., scattered photons, bremsstrahlung from beta shielding).

Deflection by fields

- Not deflected by electric or magnetic fields (no charge).

Detection

- Scintillation detectors (NaI(Tl), CsI), semiconductor detectors (Ge(Li), HPGe for high resolution), ionisation chambers, and dosimeters. Gamma spectrometry yields lines identifying isotopes via energy calibration.

Biological and safety aspects

- Deep tissue penetration \rightarrow whole-body exposures are possible and hazardous. Doses measured in Gray (Gy) for absorbed dose and Sievert (Sv) for equivalent/effective dose considering biological weighting. Shielding, distance and time are key protective measures.

Comparison Summary of Alpha, Beta and Gamma Rays

Property	Alpha (α) Rays	Beta (β) Rays	Gamma (γ) Rays
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Nature	Helium nucleus (${}^2\text{He}^4$): 2 protons + 2 neutrons	Electrons (β^-) or positrons (β^+)	Electromagnetic photon
Charge	+2e	$-1e$ (β^-) or $+1e$ (β^+)	0
Mass	Heavy (≈ 4 amu)	Very light (electron mass: $1/1836$ amu)	Zero mass
Origin	Emitted by heavy nuclei during α -decay	Emitted during β -decay (neutron \leftrightarrow proton transformation)	Emitted when excited nuclei de-excite
Energy Spectrum	Discrete (monoenergetic for each transition)	Continuous spectrum (due to neutrino)	Discrete (characteristic γ energies)
Velocity	About 1/10th of speed of light	Close to speed of light	Speed of light
Ionising Power	Very high	Moderate	Low
Penetrating Power	Very low (stopped by paper/skin)	Moderate (stopped by few mm Al)	Very high (requires thick lead/concrete)
Range in Air	Few cm	Few m	Very large (hundreds of metres)
Effect of E & B Fields	Strongly deflected (towards – plate)	Strongly deflected (β^- $\rightarrow +$ plate; $\beta^+ \rightarrow -$ plate)	Not deflected
Interaction with Matter	Heavy ionisation and rapid energy loss	Ionisation + bremsstrahlung	Photoelectric, Compton & pair production
Biological Hazard	High if inhaled/ingested; low external hazard	External & internal hazard	High external hazard
Shielding	Paper, skin	Plastic, aluminium	Lead, concrete
Detection Methods	ZnS scintillator, surface-barrier detectors	GM counter, scintillators, semiconductor detectors	NaI(Tl), HPGe detectors



Typical Examples	Uranium, thorium, radium emit α	${}^3\text{H}$, ${}^{14}\text{C}$, ${}^{60}\text{Co}$ (β), ${}^{22}\text{Na}$ (β^+)	${}^{60}\text{Co}$, ${}^{137}\text{Cs}$, ${}^{241}\text{Am}$ emit γ
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4.3 Gamow's theory of alpha decay

Introduction

Classically, an alpha particle is strongly bound inside the nucleus by a high nuclear potential well. To escape, the α -particle would need energy equal to or greater than the Coulomb barrier ($\approx 25\text{--}30\text{ MeV}$). However, experimentally the α -particle energies are only about $4\text{--}9\text{ MeV}$.

Classical mechanics cannot explain how α -particles escape the nucleus with such low energies.

Gamow (1928), independently Gurney and Condon, applied **quantum mechanics** and proposed that alpha decay occurs by **quantum tunnelling** through the nuclear potential barrier.

Basic Ideas of Gamow's Theory

1. Pre-formed alpha particle inside nucleus

Gamow assumed that:

- an α -particle exists already inside the nucleus as a “cluster”,
- it moves inside the potential well, colliding frequently with the nuclear boundary.

2. Potential barrier

The α particle inside the nucleus experiences:

1. A **deep attractive nuclear potential well** for radii less than the nuclear radius R .
2. Outside the nucleus, a **repulsive Coulomb potential** due to daughter nucleus charge.

The combined potential forms a barrier that the α -particle must cross to escape.

Potential description

- Inside the nucleus ($r < R$): attractive potential well.



- At the surface and beyond ($r > R$): repulsive Coulomb potential

$$V(r) = (2Ze^2)/(4\pi\epsilon_0 r)$$

The alpha particle energy E is less than the height of this potential barrier.

3. Quantum Tunnelling

Quantum mechanics allows a particle to penetrate and emerge through a barrier even if its energy $E <$ barrier height V .

This probability is extremely small per collision but non-zero.

The α -particle repeatedly hits the nuclear wall at very high frequency ($\sim 10^{21}$ times per second).

Even a tiny tunnelling probability yields measurable decay rates.

4. Decay Constant (λ)

(a) Frequency of collision (attempt frequency)

The alpha particle inside the nucleus behaves like a particle in a potential well.

The number of times it hits the nuclear boundary per second is called **attempt frequency**, denoted by ν .

Typical value: $\nu \approx 10^{21} \text{ s}^{-1}$

(b) Tunnelling probability (P)

Using the WKB approximation, the probability that an α -particle tunnels through the Coulomb barrier is:

$$P = \exp\left[- (2/\hbar) \int_{\text{from } R \text{ to } b} \sqrt{2m(V(r) - E)} \, dr \right]$$

where

R = nuclear radius

b = classical turning point (where $V(b) = E$)

m = mass of alpha particle

This integral gives an exponential dependence on the alpha particle energy.

(c) Decay constant

$$\lambda = \nu P$$



The half-life is related to λ by

$$T_{1/2} = \ln 2 / \lambda$$

Key Features Explained by Gamow's Theory

1. Why alpha particles escape despite insufficient classical energy.
2. Why alpha decay energies are discrete (nuclear energy levels).
3. Why small increase in α -energy leads to drastic increase in decay rate.
4. Quantitative match with the Geiger–Nuttall empirical relation.
5. Dependence of half-life on atomic number Z and energy E .

Limitations of Gamow's Theory

1. Assumes pre-formed α -particle inside nucleus (not fully justified by nuclear structure theories).
2. Treats nucleus using a simple potential model.
3. Does not include detailed nuclear many-body interactions.
4. Works best for heavy nuclei; less accurate for lighter nuclei.

4.3 Geiger Nuttall law

In alpha decay, different radioactive nuclei emit α -particles with different energies and have different half-lives.

Empirically, it was observed by **Geiger and Nuttall (1911)** that:

- Nuclei that emit **higher-energy α -particles** have **shorter half-lives**.
- Nuclei with **lower α energies** have **extremely long half-lives** (sometimes billions of years).

This relationship between **alpha particle energy (E_α)** and **decay constant (λ)** is expressed in the **Geiger–Nuttall law**.

Statement of the Geiger–Nuttall Law

The empirical form is:

$$\log_{10} \lambda = A + B (Z / \sqrt{E_\alpha})$$

or in terms of half-life:



$$\log_{10} T_{1/2} = a (Z / \sqrt{E_\alpha}) + b$$

where

Z = atomic number of daughter nucleus

E_α = energy of emitted alpha particle

A, B (or a, b) are constants for a radioactive series.

Key feature:

λ increases exponentially with $E_\alpha \rightarrow T_{1/2}$ decreases exponentially with E_α .

Alpha decay is a **quantum tunnelling** process.

- The α -particle is trapped inside the nuclear potential well.
- It must tunnel through the **Coulomb barrier** (electrostatic repulsion from daughter nucleus).
- The tunnelling probability is extremely sensitive to:
 1. **Height of the barrier** \rightarrow related to Z
 2. **Kinetic energy of α** $\rightarrow E_\alpha$
 3. **Barrier width** \rightarrow difference between nuclear radius R and classical turning point b

Thus, very small changes in E_α drastically change tunnelling probability. An α -particle of energy E (here $E \equiv E_\alpha$) is assumed to be preformed inside the parent nucleus and oscillates in a potential well. To escape it must tunnel through the Coulomb barrier outside the nuclear radius R . The Coulomb potential (for $r > R$) seen by the α and daughter nucleus of charge $+Ze$ is

$$V(r) = \frac{2Ze^2}{4\pi\epsilon_0 r} = \frac{2Zke^2}{r},$$

where $k = \frac{1}{4\pi\epsilon_0}$.

The classical (outer) turning point b is given by $V(b) = E$:

$$b = \frac{2Zke^2}{E}.$$

According to Gamow, the decay constant is

$$\lambda = \nu P,$$



where ν is the frequency of attempts (order 10^{21} s^{-1}) and P is the tunnelling probability through the barrier $R \leq r \leq b$.

In the WKB approximation the tunnelling probability is

$$P \approx \exp \left[-\frac{2}{\hbar} \int_R^b \sqrt{2m(V(r) - E)} dr \right].$$

Here m is the reduced mass of the α -particle (approximately the α mass).

Define the exponent (Gamow factor)

$$G = \frac{2}{\hbar} \int_R^b \sqrt{2m(V(r) - E)} dr,$$

so $P \approx e^{-G}$ and hence

$$\lambda = \nu e^{-G}.$$

Our task is to evaluate (or approximate) G for the Coulomb potential and show $G \propto Z/\sqrt{E}$.

Set up the integral for G

Substitute $V(r) = 2Zke^2/r$ into the integrand:

$$G = \frac{2}{\hbar} \sqrt{2m} \int_R^b \sqrt{\frac{2Zke^2}{r} - E} dr.$$

Factor out E to show the dependence on \sqrt{E} . Let E be positive; write

$$\sqrt{\frac{2Zke^2}{r} - E} = \sqrt{E} \sqrt{\frac{2Zke^2}{Er} - 1}.$$

Thus

$$G = \frac{2\sqrt{2mE}}{\hbar} \int_R^b \sqrt{\frac{2Zke^2}{Er} - 1} \frac{dr}{\sqrt{E}}.$$

A simpler useful form is

$$G = \frac{2\sqrt{2m}}{\hbar} \int_R^b \sqrt{\frac{2Zke^2}{r} - E} dr.$$



We evaluate this integral by a standard substitution.

Set

$$r = \frac{2Zke^2}{E} \cdot \frac{1}{x} \Rightarrow x = \frac{2Zke^2}{Er}.$$

Note that when $r = b$ we have $x = 1$, and when $r = R$ we have $x = x_R$ where

$$x_R = \frac{2Zke^2}{ER} = \frac{b}{R}.$$

Also $dr = -\frac{2Zke^2}{E} \frac{dx}{x^2}$. Substitute into the integrand:

$$\begin{aligned} G &= \frac{2\sqrt{2m}}{\hbar} \int_{x=x_R}^1 \sqrt{E(x-1)} \left(-\frac{2Zke^2}{E} \frac{dx}{x^2}\right) \\ &= \frac{2\sqrt{2m}}{\hbar} \cdot \frac{2Zke^2}{\sqrt{E}} \int_1^{x_R} \frac{\sqrt{x-1}}{x^2} dx. \end{aligned}$$

(we reversed limits to remove minus sign).

So

$$G = \frac{4Zke^2}{\hbar} \sqrt{\frac{2m}{E}} I(x_R), I(x_R) = \int_1^{x_R} \frac{\sqrt{x-1}}{x^2} dx.$$

This exact integral can be expressed in closed form, but for practical nuclear values $x_R = b/R \gg 1$ (because b is much larger than nuclear radius R). We therefore evaluate $I(x_R)$ approximately for large x_R .

Asymptotic evaluation for $b \gg R$ (leading term)

For $x_R \gg 1$ the dominant contribution of the integral comes from x not too close to 1; one may evaluate the integral asymptotically and obtain a simple leading term. Standard texts show that for $x_R \gg 1$,

$$I(x_R) \approx \frac{\pi}{8} \cdot 2 = \frac{\pi}{4}$$



in the leading order that enters the exponential. A more direct and commonly used and physically transparent approximation is obtained by replacing the exact integral by its leading term which yields the simple closed form

$$G \approx \frac{2\pi Zke^2}{\hbar v},$$

where v is the velocity of the emitted α -particle and $v = \sqrt{2E/m}$. To see why this form is natural note:

$$\sqrt{\frac{2m}{E}} = \frac{\sqrt{2m}}{\sqrt{E}} = \frac{\sqrt{2m}}{\sqrt{m} v} = \frac{\sqrt{2}}{v},$$

so the prefactor $\sqrt{2m/E}$ yields the $1/v$ dependence. Collecting numerical factors from the WKB integral yields the factor 2π (this is the standard Gamow result obtained after careful evaluation). Thus

$$G \approx \frac{2\pi Zke^2}{\hbar v}.$$

Substitute $v = \sqrt{2E/m}$ to show explicit dependence on E :

$$G \approx \frac{2\pi Zke^2}{\hbar} \sqrt{\frac{m}{2}} \frac{1}{\sqrt{E}} \Rightarrow G \propto \frac{Z}{\sqrt{E}}.$$

Therefore the tunnelling probability is approximately

$$P \approx e^{-G} \approx \exp\left(-\frac{2\pi Zke^2}{\hbar v}\right) = \exp\left(-\frac{2\pi Zke^2}{\hbar} \sqrt{\frac{m}{2}} \frac{1}{\sqrt{E}}\right).$$

Decay constant and logarithmic form

Recall $\lambda = \nu P$. Taking natural logarithm:

$$\ln \lambda = \ln \nu - G \approx \ln \nu - \frac{2\pi Zke^2}{\hbar} \sqrt{\frac{m}{2}} \frac{1}{\sqrt{E}}.$$

Write the coefficients as constants C_1 and C_2 :

$$\ln \lambda = C_1 - C_2 \frac{Z}{\sqrt{E}},$$



where

$$C_1 = \ln \nu, C_2 = \frac{2\pi k e^2}{\hbar} \sqrt{\frac{m}{2}}.$$

Converting to common (base-10) logarithm,

$$\log_{10} \lambda = \frac{\ln \lambda}{\ln 10} = \frac{C_1}{\ln 10} - \frac{C_2}{\ln 10} \cdot \frac{Z}{\sqrt{E}}.$$

This is the Geiger–Nuttall relation in the usual empirical linear form:

$$\log_{10} \lambda = A + B \frac{Z}{\sqrt{E}}$$

or equivalently (since $T_{1/2} = \ln 2/\lambda$),

$$\log_{10} T_{1/2} = a \frac{Z}{\sqrt{E}} + b$$

with constants A, B (or a, b) determined by ν , nuclear radius corrections and the numerical factors from the integral.

Comments on approximations and constants

1. The derivation used WKB and an asymptotic evaluation valid when $b \gg R$ (i.e. the classical turning point is far outside the nuclear radius). This is well satisfied for heavy α -emitters.
2. The factor 2π in the leading expression $G \approx 2\pi Z k e^2 / (\hbar \nu)$ emerges from the detailed evaluation of the integral; retaining more terms (nuclear radius dependent corrections) introduces additive corrections that affect the intercept A but do not change the dominant Z/\sqrt{E} dependence.
3. The pre-exponential factor ν (attempt frequency) and the small corrections from the inner part of the integral give different intercepts for different decay series; hence distinct parallel lines are observed for different radioactive families (Uranium, Thorium, Actinium series).



4. The mass m used is the α -particle mass (or reduced mass more precisely). The numerical constant C_2 depends on that mass and on fundamental constants e, \hbar, k ; when these are inserted one obtains numerical values that fit experimental slopes.

Consequences and Applications

1. Identification of isotopes

Alpha energy measurement gives idea of half-life and nuclear properties.

2. Radioactive series pattern

Isotopes belonging to a radioactive series follow straight-line behaviour when plotting:

$$\log_{10} T_{1/2} \text{ vs. } Z/\sqrt{E\alpha}$$

3. Verification of quantum tunnelling

The law is strong evidence that α -decay proceeds via **quantum mechanical tunnelling** and not classical escape.

4. Predicting unknown half-lives

Measured $E\alpha$ permits estimating half-life using known constants of the series.:

Limitations of the Geiger–Nuttall Law

1. Applies mainly to **alpha decay**, not other decay modes.
2. Different constants A, B are required for each radioactive family.
3. Requires accurate α -energy measurement.
4. Based on simplified form of tunnelling barrier.

Beta decay Spectra

Beta decay is a radioactive process in which an unstable nucleus emits:

- an electron (**β^- decay**)
- or a positron (**β^+ decay**)

and transforms a neutron \leftrightarrow proton.

Early experimental studies showed that **the energy of beta particles is not discrete** (as in α -decay), but **continuous** up to a maximum value.



This continuous energy distribution puzzled physicists, because **energy and momentum appeared not to be conserved** if only the electron/positron was emitted.

This led to a deeper understanding of nuclear decay and ultimately to the discovery of the **neutrino**.

1. The number of β particles emitted with a particular kinetic energy **is not constant**.
2. When plotted as **$N(E)$ vs E** , the distribution forms a **continuous curve** from **zero** to a **maximum endpoint energy (E_0)**.
3. No β particle is observed with energy **greater** than E_0 .
4. A large number of β particles have **intermediate energies**.

Graphical Representation

- Energy (horizontal axis): $0 \rightarrow E_0$
- Counting rate (vertical axis): Starts at 0, rises to a maximum, then falls to 0 as $E \rightarrow E_0$.

Thus, the β spectrum is **continuous**, unlike the **line spectra of α and γ rays**.

The “Energy Conservation Problem”

Before 1930, physicists believed:

- β decay emits **only the beta particle**.
- Nucleus recoils slightly, but recoil energy is very small.

If only one particle is emitted, the spectrum **must** be monoenergetic (like α -decay).

But experimentally the spectrum was **continuous**.

This suggested:

- Energy conservation seemed violated.
- Momentum conservation also seemed violated.
- Angular momentum (spin) conservation seemed violated.

This was a major theoretical crisis.

Pauli’s Neutrino Hypothesis (1930)

To restore conservation laws, Pauli proposed:

- A new, neutral, very light particle is emitted along with the β particle.

He called it the **neutrino** (Fermi later coined the name).

Properties

The neutrino must:



1. Carry away part of the decay energy.
2. Have nearly zero mass.
3. Have no charge.
4. Interact very weakly with matter.

Thus, the **total decay energy (Q value)** is shared between:

- the β particle
- the neutrino
- the recoil nucleus (very small share)

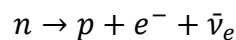
This naturally produces a **continuous β -energy spectrum**.

Fermi's Theory of Beta Decay

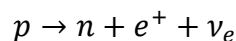
Fermi (1934) formulated the first **quantum mechanical theory** of β decay.

He proposed that:

- β decay is due to the **weak interaction**.
- The neutron transforms into a proton by emitting an electron and an antineutrino:



or in β^{+} decay:



Energy Sharing

Let the total decay energy be **Q**.

Then energy conservation requires:

$$Q = E_e + E_\nu + E_R$$

Where:

- E_e = electron kinetic energy
- E_ν = neutrino energy
- E_R = recoil energy of daughter nucleus

Since E_R is very small:

$$Q \approx E_e + E_\nu$$

Depending on how these two particles share energy, the β electron can have:



- Very low energy
- Intermediate energy
- Maximum energy (if neutrino carries nearly zero kinetic energy)

Thus, the **continuous β spectrum** is fully explained.

Mathematical Form of Beta Spectrum

Fermi derived the probability that the electron has momentum between p and $p + dp$:

$$N(p) dp \propto p^2 (E_0 - E_e)^2 dp$$

Where:

- E_0 = endpoint energy
- E_e = kinetic energy of β particle
- p = electron momentum

Using relativistic relation:

$$E_e = \sqrt{p^2 c^2 + m_e^2 c^4} - m_e c^2$$

This gives the **shape** of β spectrum.

Endpoint Energy and Q-Value

The **maximum kinetic energy** of β particle is:

$$E_{\max} = Q$$

This occurs when neutrino takes ~ 0 kinetic energy.

Thus:

- Endpoint of the β spectrum gives **Q value of the decay**.
- Continuous curve from $0 \rightarrow Q$.

Experimental Feature	Explained by Neutrino Theory
Continuous β spectrum	Energy shared between e^- and ν
Conservation of energy	Restored
Conservation of momentum	Restored
Conservation of spin	Neutrino spin (1/2) balances spin values



Shape of β spectrum	Derived quantitatively by Fermi
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Unit 5: Nuclear Reactor, Accelerator and Detectors

1. Nuclear Fission
2. Nuclear reactor
3. radio isotopes and its applications
4. Charged particle accelerators
5. Detectors of Nuclear radiation
6. Geiger Muller Counter
7. Scintillation Counter
8. Nuclear Facilities in India

5.1 Nuclear Fission

Nuclear fission is a fundamental nuclear process in which a **heavy, unstable atomic nucleus splits into two lighter nuclei**, releasing an enormous amount of energy in the process. This phenomenon was first observed in 1938 by Otto Hahn and Fritz Strassmann and later explained by Lise Meitner and Otto Frisch. Fission typically occurs when nuclei such as **Uranium-235** or **Plutonium-239** absorb a **slow (thermal) neutron**, becoming highly unstable and breaking apart.

The energy released in nuclear fission comes mainly from the **conversion of mass into energy**, as described by Einstein's equation $E = mc^2$. Along with energy, the reaction also emits **additional neutrons**, which can trigger further fission events—leading to what is known as a **chain reaction**. When this chain reaction is carefully regulated, it becomes the basis of modern **nuclear reactors**, providing a major source of electricity. When it is left uncontrolled, it results in the devastating explosive energy of a **nuclear weapon**.

Thus, nuclear fission stands as one of the most significant discoveries of the 20th century, shaping both the peaceful use of nuclear energy and the development of nuclear arms, and continues to remain a key topic in nuclear physics and energy science.

The discovery of nuclear fission dates back to **1938**, when **Otto Hahn** and **Fritz Strassmann** detected the presence of lighter elements such as barium after bombarding



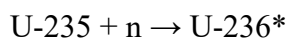
uranium with neutrons. Later, **Lise Meitner** and **Otto Frisch** interpreted this as the splitting of the uranium nucleus. Their explanation laid the foundation for a new understanding of nuclear structure and stability.

Nuclear fission generally involves **heavy nuclei** such as:

- Uranium-235 (U-235)
- Uranium-233 (U-233)
- Plutonium-239 (Pu-239)

These nuclei are **neutron-rich** and unstable in microscopic terms, but they do not undergo spontaneous fission readily. However, when such a nucleus absorbs a **slow (thermal) neutron**, it becomes highly unstable and forms a short-lived, excited compound nucleus.

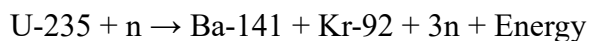
For example:



The compound nucleus **U-236*** is extremely unstable and immediately splits into two lighter nuclei, releasing:

- **Fission fragments** (medium-mass nuclei, often around mass numbers 90–140)
- **2 or 3 neutrons**
- **A large amount of energy**
- **Gamma radiation**

A typical fission reaction is:



The energy released in fission arises due to the **mass defect**. The combined mass of the fission products and emitted neutrons is **less** than the original mass of the U-235 nucleus plus the neutron. This missing mass is converted into energy according to Einstein's equation:

$$E = mc^2$$

A single U-235 fission event releases about **200 MeV** of energy, which is many millions of times greater than the energy released in ordinary chemical reactions.

Energy appears in the form of:

- Kinetic energy of fission fragments
- Kinetic energy of emitted neutrons
- Gamma radiation
- Beta and gamma rays from radioactive decay of fission fragments



The neutrons released during fission can induce further fission in other U-235 nuclei. This leads to a **chain reaction**.

There are two types of chain reactions:

(a) Controlled Chain Reaction

Used in **nuclear reactors**

- Controlled by inserting materials like **boron** or **cadmium rods**, which absorb excess neutrons.
- Ensures that exactly **one neutron** from each fission event causes another fission.
-

(b) Uncontrolled Chain Reaction

Occurs in **atomic bombs**

- No control rods or moderators
- Reaction multiplies rapidly
- Leads to an explosive release of energy within fractions of a second

For sustained fission, the following conditions must be satisfied:

1. **Presence of fissile material** (U-235, Pu-239)
2. **Availability of slow neutrons**, as U-235 has a higher fission probability for low-energy neutrons
3. **Sufficient quantity of material**, known as the **critical mass**
4. **Use of a moderator** (in reactors), such as heavy water or graphite, to slow down fast neutrons
5. **Proper arrangement of fuel, moderator, and neutron absorber**

The fission fragments produced are usually:

- Highly unstable
- Radioactive
- Beta-emitters

These fragments undergo a series of radioactive decays until stable isotopes are formed. This is one reason why **nuclear waste remains radioactive for long periods**.

(a) Induced Fission



Fission initiated by bombarding nuclei with neutrons. Most nuclear energy systems rely on induced fission.

(b) Spontaneous Fission

Fission occurring naturally without neutron absorption.

Occurs rarely in heavy elements like uranium and thorium.

Applications of Nuclear Fission

1. **Nuclear Power Plants** – Electricity generation
2. **Nuclear Weapons** – Uncontrolled chain reaction
3. **Radioisotope Production** – Medical and industrial applications
4. **Nuclear Submarines and Ships** – Compact, long-lasting energy source

Advantages and Disadvantages of Fission

Advantages

- Produces large amounts of energy
- No greenhouse gases during operation
- Requires small fuel quantities

Disadvantages

- Produces long-lasting radioactive waste
- High risk if safety systems fail
- High installation and maintenance costs
- Possibility of nuclear weapon proliferation

5.2 Nuclear reactor

A nuclear reactor operates on the principle of **controlled nuclear fission**. When fissile materials such as **Uranium-235** or **Plutonium-239** absorb slow neutrons, they undergo fission, releasing:

- A large amount of energy
- 2 or 3 fast neutrons
- Fission fragments
- Gamma radiation



Figure 5. 1 (Nuclear Reactor)



The fast neutrons are slowed down using a **moderator**, ensuring that they are more likely to cause further fission events. By regulating the number of neutrons available for fission, the reactor maintains a stable and self-sustaining **chain reaction**.

Essential Components of a Nuclear Reactor

(a) Fuel

Fissile materials that undergo fission.

Common reactor fuels:

- Uranium-235
- Uranium-233
- Plutonium-239

Fuel is usually made into **fuel rods** or **fuel pellets**, arranged in assemblies.

(b) Moderator

A substance used to **slow down fast neutrons** to thermal energies, increasing the probability of fission.

Common moderators include:

- **Heavy water (D_2O)**
- **Light water (H_2O)**
- **Graphite**

(c) Control Rods

Made of neutron-absorbing materials such as **boron**, **cadmium**, or **hafnium**.

Functions:

- Regulate the rate of chain reaction
- Shut down the reactor in emergencies

By inserting control rods deeper, more neutrons are absorbed → reaction slows.

By withdrawing them, more neutrons are available → reaction speeds up.

(d) Coolant

Removes heat from the reactor core and transfers it to the heat exchanger or turbine.

Common coolants:



- Water
- Heavy water
- Liquid sodium
- Helium gas

Coolants must have:

- High heat capacity
- Chemical stability
- Low neutron absorption

(e) Reactor Core

The central part of the reactor containing fuel rods, moderator, and control rods.

It is the region where **fission reactions occur** and heat is generated.

(f) Pressure Vessel

A strong steel container that encloses the reactor core.

It withstands high pressure and high temperatures.

(g) Biological Shield

Thick concrete walls around the reactor prevent harmful **neutron and gamma radiation** from escaping into the environment.

(h) Heat Exchanger / Steam Generator

Transfers heat from the coolant to water, producing **steam** that drives turbines connected to electric generators.

(i) Turbine and Generator

- Steam rotates the turbine blades
- Turbine drives the generator
- Generator produces electricity

This process is similar to thermal power plants, except the heat source is nuclear.

Working of a Nuclear Reactor (Step-by-Step)

1. **Fission starts** when slow neutrons strike fissile fuel nuclei inside the reactor core.
2. The fission reaction releases heat and more neutrons.



3. The **moderator** slows the neutrons, sustaining further fission.
4. **Control rods** adjust the number of neutrons to maintain a steady reaction rate.
5. The **coolant** absorbs heat from the core and carries it to the heat exchanger.
6. Heat converts water into **steam**.
7. Steam drives the **turbine**, producing mechanical energy.
8. The **generator** converts mechanical energy into electrical energy.
9. Steam is condensed back to water and reused, forming a closed cycle.

Types of Nuclear Reactors

(1) Pressurized Water Reactor (PWR)

- Uses light water as moderator and coolant
- Water remains at high pressure to prevent boiling

(2) Boiling Water Reactor (BWR)

- Water boils inside the reactor core
- Steam is generated directly in the core

(3) Heavy Water Reactor (HWR) / CANDU Reactor

- Uses heavy water as coolant and moderator
- Can use natural uranium as fuel

(4) Fast Breeder Reactor (FBR)

- Uses fast neutrons (no moderator)
- Produces more fissile material (Pu-239) than it consumes
- Uses liquid sodium as coolant

(5) Gas-Cooled Reactor

- Moderator: graphite
- Coolant: carbon dioxide or helium

Advantages of Nuclear Reactors

- High energy output from small amount of fuel
- No emission of greenhouse gases during operation
- Reliable and continuous power generation



- Useful for producing radioisotopes
- Long-term fuel supply

Disadvantages and Challenges

- Radioactive waste disposal
- High installation and maintenance cost
- Risk of radiation leak or reactor meltdown
- Requires highly trained personnel
- Possibility of nuclear proliferation

Applications of Nuclear Reactors

1. Electricity generation
2. Nuclear submarines and naval propulsion
3. Medical isotope production
4. Industrial radiography
5. Research in nuclear physics

5.3 Radio isotopes and its applications

A **radioisotope** (or radioactive isotope) is an isotope of an element that has an **unstable nucleus**. Because of this instability, the nucleus undergoes **radioactive decay**, emitting radiation in the form of **alpha, beta, or gamma rays**. Radioisotopes may occur **naturally** (e.g., Carbon-14, Potassium-40, Uranium-238) or be **artificially produced** in nuclear reactors and particle accelerators (e.g., Cobalt-60, Iodine-131, Technetium-99m). These isotopes have wide applications due to their **radioactive nature, known half-life, and predictable decay patterns**, making them valuable tools in medicine, industry, agriculture, archaeology, and scientific research.

Characteristics of Radioisotopes



1. **Unstable Nucleus:**

Tends to attain stability by emitting radiation.

2. **Definite Half-Life:**

Time required for half of the radioactive atoms to decay. This property allows accurate age determination and dosage calculation.

3. **Type of Emission:**

Each isotope emits characteristic radiation (α , β , or γ).

4. **Detectability:**

Radiation can be detected using instruments like **Geiger-Müller counters**, **scintillation counters**, etc.

5. **Penetration and Ionization:**

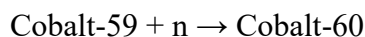
Different radiations have different penetrating powers, which allows specific applications.

Radioisotopes are produced mainly by:

(a) Neutron Activation (most common)

A stable nucleus absorbs a neutron and becomes radioactive.

Example:



(b) Particle Bombardment

Using protons, deuterons, or alpha particles in particle accelerators.

(c) Fission Products

Some radioisotopes are obtained as fission fragments from nuclear reactors.

Medical Applications

(a) Diagnosis

1. **Iodine-131**

Used to detect thyroid disorders, hyperthyroidism, and thyroid cancer.

2. **Technetium-99m**

Used in imaging of brain, liver, kidneys, and bone.

It is the most widely used diagnostic tracer due to its short half-life and pure gamma emission.

3. **Sodium-24**

Used to study blood circulation rate.



(b) Therapy

1. **Cobalt-60**

Used in radiotherapy to destroy cancerous cells.

2. **Phosphorus-32**

Used to treat blood cancers such as leukemia.

3. **Iodine-131 (Therapeutic dose)**

Used to treat hyperthyroidism and thyroid cancer.

Why radioisotopes are useful in medicine?

- They can trace biochemical pathways.
- They selectively accumulate in specific organs.
- Short half-life → less radiation exposure.

Industrial Applications

(a) Non-Destructive Testing (NDT)

Gamma rays from **Cobalt-60** or **Iridium-192** detect cracks and defects in metal structures, pipelines, and aircraft components.

(b) Thickness Gauging

Beta emitters like **Strontium-90** measure the thickness of paper, plastics, rubber sheets, and metal foils.

(c) Leak Detection

Radioactive tracers are used to locate leaks in oil pipelines and water channels.

(d) Sterilization

Gamma radiation from **Cobalt-60** sterilizes medical instruments, syringes, bandages, and food materials.

Agricultural Applications

(a) Mutation Breeding

Gamma rays (Co-60) induce mutations to create high-yielding, disease-resistant crops.



(b) Food Preservation (Food Irradiation)

Used to destroy microbes and insects in food grains, fruits, and vegetables, increasing storage life.

(c) Tracer Studies

Radioisotopes like **Phosphorus-32** and **Carbon-14** are used to study nutrient uptake, photosynthesis, and metabolic pathways in plants.

Archaeological and Geological Applications

(a) Radiocarbon Dating (Carbon-14)

Used to estimate the age of archaeological artifacts, fossils, wood, bones, and cloth up to 50,000 years old.

(b) Uranium-Lead Dating

Determines the age of rocks and minerals, often millions to billions of years old.

(c) Potassium-Argon Dating

Used for volcanic rocks.

Scientific and Research Applications

1. **Radioactive tracers** help scientists follow chemical reactions.
2. **Neutron activation analysis** identifies trace elements in materials.
3. **Radiolabeling** helps in molecular biology and genetic studies.
4. **Study of reaction mechanisms** using tagged atoms.

Advantages of Radioisotopes

- Highly sensitive detection
- Can trace processes that are otherwise invisible
- Useful in medicine without major surgery
- Enable non-destructive testing
- Improve agricultural productivity
- Provide accurate dating methods

Disadvantages / Hazards



- Radiation exposure can be harmful
- Requires careful storage and disposal
- High cost of production
- Strict safety protocols needed in hospitals and industry

Radioisotopes play a crucial role in modern science and technology. Their unique properties—especially radioactive decay and measurable half-lives—enable indispensable applications in medicine, industry, agriculture, archaeology, and research. While they must be handled with strict safety measures, the benefits they offer in diagnosis, treatment, testing, and scientific discovery are enormous and irreplaceable.

5.4 Charged particle accelerators

A **charged particle accelerator** is a scientific device used to increase the kinetic energy of charged particles such as **electrons, protons, and ions** to very high speeds by applying **electric and/or magnetic fields**. These accelerated particles are then used for studying nuclear reactions, producing radioisotopes, investigating the structure of matter, and conducting various research applications in physics, medicine, and industry. Charged particle accelerators play a fundamental role in modern nuclear physics and high-energy research.

Principle of Operation

Charged particles gain energy when they are subjected to:

- **Electric fields** → increase the speed (acceleration)
- **Magnetic fields** → change direction (circular/spiral motion)

A repeated application of accelerating electric fields allows the particles to reach extremely high velocities, sometimes close to the speed of light.

The key principle is:

Force on charged particle:

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

- **E-field** accelerates



- **B-field** guides and bends the beam

Charged particle accelerators are mainly classified into:

1. **Linear Accelerators (Linacs)**
2. **Circular Accelerators**
 - Cyclotron
 - Synchrotron
 - Betatron

Each type uses different methods to accelerate particles efficiently.

Linear Accelerator (Linac)

A **linear accelerator (linac)** accelerates charged particles in a **straight line** using a series of **drift tubes** or cavities connected to a high-frequency alternating voltage.

Working Principle

1. Particles are injected at one end.
2. Alternating electric fields accelerate the particles each time they pass between drift tubes.
3. As particles gain speed, tube lengths increase to match their velocity.
4. The final particle beam emerges with very high kinetic energy.

Applications

- Cancer therapy (electron linacs)
- Producing high-energy electrons and protons
- Injecting particles into larger circular accelerators

Cyclotron

A **cyclotron** is a circular charged particle accelerator invented by Ernest Lawrence.

It uses:

- **A constant magnetic field** (to bend the particles into a circular path)
- **An alternating electric field** (to accelerate the particles)

Structure

- Two hollow semicircular chambers called **Dees (D-shaped)**
- Strong magnetic field perpendicular to the Dees



- Oscillating electric field between the Dees
- Ion source at the center

Working

1. Charged particles start at the center.
2. Electric field accelerates them each time they cross the gap between the Dees.
3. Magnetic field forces them into a circular path.
4. As speed increases, particles move in larger circular orbits.
5. They spiral outward and are extracted with high energy.

Applications

- Producing radioisotopes (e.g., for PET scans)
- Proton and alpha-particle beams for nuclear experiments
- Medical treatments (proton therapy)

Synchrotron

A **synchrotron** is a circular accelerator where both the **magnetic field** and **electric field frequency** are varied (synchronized) with the increasing energy of particles.

Key Features

- Particles move in a fixed circular path
- Magnetic field strength increases as particle energy increases
- Used for extremely high particle energies, even up to TeV scale

Applications

- Large-scale research (e.g., CERN accelerators)
- Synchrotron radiation sources
- Studying atomic and molecular structures

Betatron

A **betatron** accelerates **electrons** using **changing magnetic fields**, exploiting **electromagnetic induction**.

Working Principle

- A time-varying magnetic field induces an electric field (Faraday's law)
- Induced electric field accelerates electrons in a circular orbit



- Magnetic field also maintains the circular path

Applications

- High-energy X-ray production
- Industrial radiography
- Radiation therapy

Need for Charged Particle Accelerators

Charged particle accelerators are essential in:

1. **Nuclear physics research**
 - Studying nuclear structure
 - Initiating nuclear reactions
2. **High-energy particle physics**
 - Understanding fundamental particles
 - Discovery of quarks, Higgs boson, etc.
3. **Medical applications**
 - Cancer radiotherapy
 - Producing medical isotopes
4. **Industrial applications**
 - Sterilization
 - Material modifications
5. **Scientific experiments**
 - X-ray scattering
 - Semiconductor research

Advantages of Particle Accelerators

- Provide controlled high-energy beams
- Enable creation of new isotopes
- Used for medical imaging and treatment
- Essential for probing subatomic particles
- Non-destructive testing using accelerator-based radiation

Limitations

- Extremely expensive to build and maintain



- Require sophisticated technology and power
- High radiation hazards if not properly shielded
- Large installations (especially synchrotrons)

Charged particle accelerators are vital instruments in modern scientific research. By accelerating particles to high speeds and directing them toward targets, they enable the exploration of atomic and subatomic phenomena, medical advancements, industrial innovations, and development of new materials. Their contribution to nuclear physics, particle physics, and applied science is unparalleled.

5.5 Detectors of Nuclear radiation

Nuclear radiations such as **alpha (α), beta (β), and gamma (γ) rays** are invisible and cannot be detected by human senses. Hence, special instruments called **radiation detectors** are used to detect, measure, and study these radiations. These detectors work based on the **ionization, scintillation, or electrical effects** produced by radiation in matter. Radiation detectors are essential in nuclear physics experiments, medical diagnostics, radiotherapy, industry, and environmental monitoring.

Classification of Nuclear Radiation Detectors

Radiation detectors may be broadly classified into:

1. **Gas-filled Detectors**
 - Ionization Chamber
 - Proportional Counter
 - Geiger-Müller (GM) Counter
2. **Scintillation Detectors**
3. **Solid-State (Semiconductor) Detectors**
 - Silicon detector
 - Germanium detector

Each detector works on different physical principles and is suited to specific types of radiation.

Gas-Filled Detectors



Gas-filled detectors contain an inert gas (argon, neon, helium) at low pressure inside a chamber with two electrodes.

When nuclear radiation passes through the gas, it **ionizes** the gas molecules.

The ions and electrons produced are collected by an electric field, giving a measurable **electrical pulse**.

Ionization Chamber

Principle

Works on the **direct measurement of ionization** produced by radiation.

A small voltage is applied so that ions produced are collected without further multiplication.

Construction

- Metal chamber filled with gas
- Central positive electrode (anode)
- Outer negative electrode (cathode)
- Ammeter to measure ion current

Working

- Radiation enters the chamber and ionizes gas
- Electrons move to anode, positive ions to cathode
- The collected charge gives a current proportional to the intensity of radiation

Applications

- Measurement of high radiation dose rates
- Monitoring radiation in nuclear reactors
- Calibration of other detectors

Proportional Counter

Principle

Uses **gas multiplication**: secondary ionization occurs near the anode wire → produces pulse proportional to the energy of incident radiation.

Construction

- Cylindrical tube filled with inert gas



- Thin central anode wire
- High voltage supply (higher than ionization chamber)

Working

- Radiation ionizes gas
- Electrons accelerate rapidly and create further ionization
- Results in proportional output pulse

Applications

- Detecting α and β particles
- Energy measurement of radiation
- Neutron detection (with BF_3 gas)

Semiconductor Detectors

These detectors use materials such as **silicon** or **germanium**.

Radiation creates **electron–hole pairs** in the semiconductor.

An electric field collects these charges to produce a pulse.

Types

- **Silicon (Si) detector:** For α and β particles
- **Germanium (Ge) detector:** For γ -ray spectroscopy (Ge(Li), HPGe detectors)

Working

1. Radiation interacts with semiconductor
2. Creates thousands of electron–hole pairs
3. Electric field collects charges
4. Signal proportional to energy is obtained

Advantages

- Excellent energy resolution
- Compact in size
- Used for precise measurements

Applications

- Gamma-ray spectroscopy
- Nuclear research



- Radiation monitoring

Cloud Chamber (Historical Detector)

Principle

Radiation ionizes supersaturated vapor.

The ionized trails condense water droplets → visible tracks.

Applications

- Observing tracks of α and β particles
- Studying cosmic rays

Bubble Chamber (Historical Detector)

Principle

Superheated liquid boils along ionization tracks → bubble trails formed.

Applications

- High-energy particle physics
- Studying particle interactions

Though not used today, they played a major role in earlier discoveries.

Applications of Radiation Detectors

1. **Nuclear research** (particle identification, spectroscopy)
2. **Medical applications**
 - PET
 - CT using radioactive tracers
 - Radiotherapy monitoring
3. **Industrial testing**
 - NDT, radiography
4. **Environmental radiation monitoring**
5. **Security and customs scanning**
6. **Radioactive contamination detection**

Detectors of nuclear radiation are essential tools for measuring, analyzing, and understanding radioactive processes. By using different physical principles—ionization, scintillation, or semiconductor effects—various detectors offer different levels of sensitivity



and accuracy. Their applications span across scientific research, industry, medical diagnosis, and environmental safety.

5.6 Geiger Muller Counter

The Geiger–Müller Counter is one of the most widely used instruments for detecting and measuring nuclear radiation. It is particularly efficient for detecting ionizing radiations such as alpha (α), beta (β), and gamma (γ) rays. The device was developed by Hans Geiger and Walther Müller in 1928 as an improvement over the earlier Geiger tube. Because of its simplicity, portability, and high sensitivity, the GM counter remains a standard tool in nuclear laboratories, hospitals, industries, and environmental monitoring.

Construction

A Geiger–Müller counter mainly consists of a cylindrical metal chamber acting as the cathode, and a thin central wire stretched along the axis of the cylinder acting as the anode.

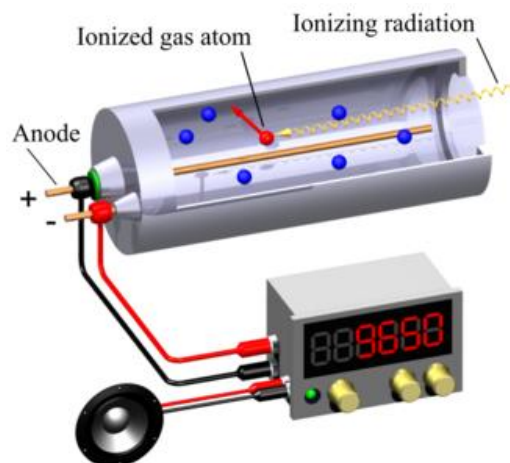


Figure 5. 2 Geiger–Müller counter

1. The chamber is filled with an inert gas such as argon or helium at low pressure (around 100 mm Hg).
2. A quenching gas such as ethyl alcohol or halogens is added in small amounts to prevent continuous discharge.
3. A high voltage supply (typically between 400 V and 1000 V) is connected between the anode and cathode.



4. A thin mica window is provided at one end of the tube to allow low-energy alpha and beta particles to enter the chamber.
5. The electrical pulses produced in the tube are fed into a counting device or scaler, which records the number of pulses.

Working Principle

The working of a GM counter is based on the ionization produced by radiation in the gas-filled chamber. When an ionizing particle enters the tube:

1. It collides with the gas molecules and produces ion pairs (positive ions and electrons).
2. The strong electric field between the electrodes accelerates these electrons towards the anode, causing them to gain enough energy to ionize more gas molecules. This leads to an avalanche of electrons.
3. The avalanche results in a large, sudden pulse of current in the external circuit.
4. Each incident particle produces a pulse of nearly the same magnitude, regardless of the original particle's energy. This means the GM counter cannot distinguish between different types or energies of radiation; it only counts the number of particles.
5. The quenching gas absorbs excess energy and prevents continuous discharge, ensuring the counter resets quickly for the next detection event.

Characteristics

1. The GM counter operates in a region known as the Geiger plateau, where the pulse height remains nearly constant over a wide voltage range.
2. It produces very large electrical pulses, making it easy to interface with simple counting circuits.
3. There is a finite dead time (about 200–300 microseconds) during which the detector cannot record another particle because the ions must recombine.

Applications

1. Measuring environmental radiation levels and background radiation.
2. Detecting radioactive contamination in laboratories, hospitals, and nuclear plants.
3. Monitoring radiation exposure for workers in nuclear installations.
4. Used in educational laboratories for basic nuclear experiments.
5. Checking leakage from sealed radioactive sources.

6. Survey meters used by first responders often have GM tubes for rapid field measurements.

Advantages

1. Simple construction and easy operation.
2. Very sensitive to beta and gamma rays.
3. Provides clear, audible or visual signals for each radiation event.
4. Portable and relatively inexpensive.

Limitations

1. Cannot differentiate between different types of radiation.
2. Cannot measure energy of the incident particles.
3. Limited accuracy at very high radiation levels because of dead time.
4. Alpha detection is possible only when a thin mica window is present.

5.1 Scintillation Counter

The scintillation counter is an advanced and highly sensitive instrument used for detecting and measuring nuclear radiation. It operates based on the principle that certain materials emit small flashes of light, called scintillations, when struck by ionizing radiation.

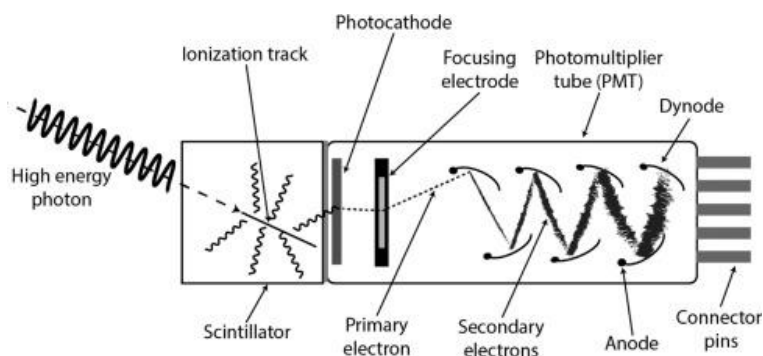


Figure 5. 3

These tiny flashes are converted into electrical signals and measured. Scintillation counters are widely used in nuclear physics, medical imaging, environmental monitoring, and high-energy research because of their excellent sensitivity and fast response.



Principle

When an ionizing particle such as an alpha, beta, or gamma ray passes through a scintillating material, it excites the atoms or molecules of the material. When these atoms return to their ground state, they emit photons of visible or ultraviolet light. The amount of light produced is proportional to the energy deposited by the particle. This light is collected and amplified using a photomultiplier tube (PMT), which converts light pulses into measurable electrical pulses.

Thus, the scintillation counter works on three basic processes:

1. Production of scintillation light.
2. Conversion of light into an electrical signal.
3. Amplification and measurement of the electrical signal.

Construction

A typical scintillation counter consists of the following components:

1. Scintillator:

A material that emits light when exposed to radiation. Scintillators are of two types:

- (a) Organic scintillators – anthracene, stilbene, plastic scintillators.
- (b) Inorganic scintillators – sodium iodide doped with thallium (NaI(Tl)), cesium iodide (CsI), bismuth germanate (BGO).

Inorganic crystals like NaI(Tl) are most commonly used for gamma-ray detection.

2. Light guide:

A transparent tube or optical material that transfers the emitted light from the scintillator to the photomultiplier tube.

3. Photomultiplier tube (PMT):

A highly sensitive device that converts light pulses into electrical pulses. It consists of a photocathode, a series of dynodes for electron multiplication, and an anode to collect the amplified electrons.

4. High voltage supply:

Provides the necessary potential difference (typically 500–1500 V) for the photomultiplier tube to operate.



5. Pulse amplifier and electronic circuit:

These sections amplify the weak electrical signals and shape them into pulses suitable for counting and measurement.

6. Counting system or multichannel analyzer:

Records the number of pulses or analyses the pulse height to determine the energy of the incident radiation.

Working

1. The incoming radiation interacts with the scintillator and produces light flashes.
2. These light flashes travel through the light guide and strike the photocathode of the PMT.
3. The photocathode releases electrons due to the photoelectric effect.
4. These electrons are accelerated and multiplied through successive dynodes, resulting in a large number of electrons at the anode.
5. The final electrical pulse is proportional to the intensity of the scintillation light, and hence, to the energy of the radiation.
6. The counting system records the number of pulses or analyses their heights to identify the type and energy of radiation.

Types of Scintillation Counters

1. Organic scintillation counters – suitable for beta particles and fast neutrons.
2. Inorganic scintillation counters – especially suitable for gamma-ray spectroscopy.
3. Liquid scintillation counters – used for detecting very low-energy beta emitters like tritium (H-3) and carbon-14.

Advantages

1. High sensitivity and fast response time.
2. Capable of measuring both the presence and energy of radiation.
3. Suitable for gamma-ray spectroscopy due to energy resolution.
4. Can detect very low levels of radioactivity.
5. Suitable for medical imaging devices such as PET and SPECT.

Limitations

1. Expensive compared to simpler detectors like GM counters.
2. Inorganic crystals like NaI(Tl) are hygroscopic and require air-tight enclosures.
3. Performance is affected by temperature and mechanical vibrations.



4. Requires careful calibration and stable high-voltage supply.

Applications

1. Gamma-ray spectroscopy and energy measurement.
2. Medical imaging (PET, SPECT scanners).
3. Environmental radiation monitoring.
4. Detection of low-energy beta emitters in biological samples.
5. High-energy physics research and cosmic ray studies.
6. Radiation protection and contamination surveys.

Feature	Ionization Chamber	Geiger–Müller Counter	Scintillation Counter
Principle	Measures ionization current produced directly by radiation in a gas	Detects ionizing radiation through gas amplification (avalanche effect)	Detects light flashes from scintillator and converts them into electrical pulses
Sensitivity	Low	Moderate to high	Very high
Pulse Size	Very small; proportional to energy	Large pulses of same size; independent of energy	Pulse size proportional to energy of radiation
Energy Measurement	Possible but limited accuracy	Not possible (all pulses same height)	Excellent energy resolution (especially NaI(Tl))
Detectable Radiation	Alpha, beta, gamma (with low efficiency)	Good for beta and gamma; limited for alpha	Detects alpha, beta, gamma, neutrons (depending on scintillator)
Operational Voltage	Low (100–300 V)	High (400–1000 V)	Moderate to high (500–1500 V)
Dead Time	Very low	High (200–300 μ s)	Very low
Response Time	Slow	Moderate	Very fast



Applications	Dosimetry, survey meters, gamma monitoring	Radiation survey, contamination check	Gamma spectroscopy, PET/SPECT, research labs
Cost	Low	Moderate	High
Energy Resolution	Poor	None	Good to excellent
Construction	Simple	Simple	Complex (requires PMT, scintillator, electronics)

5.1 Nuclear Facilities in India

India has developed a strong and comprehensive nuclear infrastructure for research, power generation, fuel production, waste management, and strategic applications. These facilities operate under the Department of Atomic Energy (DAE), Government of India. The nuclear programme of India follows a three-stage strategy proposed by Dr. Homi J. Bhabha, focusing on optimal utilization of India's uranium and thorium resources. The major nuclear facilities include research reactors, nuclear power plants, fuel processing centres, heavy water plants, and waste management units.

1. Research Reactors

These reactors are mainly used for research, isotope production, neutron studies, and training.

1. Apsara – India's first research reactor, commissioned in 1956 at Bhabha Atomic Research Centre (BARC), Mumbai.
2. CIRUS – A 40 MW research reactor at BARC (in operation from 1960 to 2010).
3. Dhruva – A high-flux research reactor (100 MW) operating at BARC since 1985.
4. Kamini – A unique 30 kW reactor at Indira Gandhi Centre for Atomic Research (IGCAR), Kalpakkam, using uranium-233 fuel.



2. Nuclear Power Plants

Nuclear power plants generate electricity using various types of reactors. India primarily uses Pressurized Heavy Water Reactors (PHWRs), Boiling Water Reactors (BWRs), and is developing Fast Breeder Reactors (FBRs).

Major operational nuclear power stations include:

1. Tarapur Atomic Power Station (Maharashtra) – India's first nuclear power plant.
2. Rajasthan Atomic Power Station (RAPS), Kota.
3. Madras Atomic Power Station (MAPS), Kalpakkam.
4. Kaiga Generating Station (Karnataka).
5. Narora Atomic Power Station (Uttar Pradesh).
6. Kakrapar Atomic Power Station (Gujarat).
7. Kudankulam Nuclear Power Plant (Tamil Nadu) – built with Russian collaboration, uses VVER reactors.
8. Upcoming units include Jaitapur (Maharashtra), Kovvada (Andhra Pradesh), and Gorakhpur (Haryana).



Figure 5. 4 (Kudankulam Nuclear Power Plant)

3. Fuel Cycle Facilities

India has developed independent facilities for uranium mining, processing, and fuel fabrication.

1. Uranium Corporation of India Ltd. (UCIL) – mines uranium at Jaduguda, Turamdih, Narwapahar (Jharkhand).
2. Nuclear Fuel Complex (NFC), Hyderabad – fabricates fuel assemblies for PHWRs and BWRs.
3. Heavy Water Board (HWB) – operates heavy water plants at Kota, Manuguru, Thal, Hazira, and Tuticorin.



4. Fast Breeder and Advanced Reactors

India is a global leader in fast breeder reactor technology.

1. Prototype Fast Breeder Reactor (PFBR) – located at Kalpakkam; uses mixed oxide (MOX) fuel.
2. Future stage-II reactors aim to convert uranium-238 into plutonium-239.
3. Thorium-based reactors (stage-III) are under development, focusing on converting thorium-232 to uranium-233.

5. Reprocessing and Waste Management

India has advanced technology for reprocessing spent nuclear fuel.

1. Reprocessing plants at Trombay, Tarapur, and Kalpakkam.
2. Waste immobilization plants at Tarapur and Trombay.
3. Radioactive waste is processed, vitrified, and stored safely.

6. Particle Accelerators and Research Centres

1. Variable Energy Cyclotron Centre (VECC), Kolkata – operates cyclotrons for nuclear and particle physics research.
2. Raja Ramanna Centre for Advanced Technology (RRCAT), Indore – specializes in lasers and particle accelerators.
3. Tata Institute of Fundamental Research (TIFR), Mumbai – research in nuclear, particle, and cosmic ray physics.

7. Regulatory Bodies

1. Atomic Energy Commission (AEC) – apex policy-making body.
2. Atomic Energy Regulatory Board (AERB) – ensures safety of all nuclear facilities.
3. Nuclear Power Corporation of India Limited (NPCIL) – responsible for nuclear power plant operation.
4. Bharatiya Nabhikiya Vidyut Nigam Limited (BHAVINI) – operates fast breeder reactors.



8. Applications and Contributions

1. Production of radioisotopes for medicine, agriculture, and industry.
2. Electricity generation for national grid.
3. Research in nuclear physics, radiation biology, and material science.
4. Development of advanced reactor technology and thorium utilisation.
5. Contribution to national security and strategic programmes.